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The Spectrum of MgF

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Measurements of the band heads and of the partially resolved rotational structure of the MgF absorption bands are used to evaluate the following constants for the $^2\Sigma$, $^2\Sigma$ system: $\nu_{00}=37{,}187.4$ cm^{-1}, $B_0'=0.537$, $B_0''=0.518\pm0.010$, $r_0''=1.75\times10^{-8}$ cm and for the $^2\Pi$, $^2\Sigma$ system: $\nu_{00}=27{,}846.5$, $B_0'=0.529$, $B_0''=0.518$, $A=-34.3$ (or $+38.3$). The isotope effect of Mg is observed in the $+1$ sequence of $^2\Sigma$, $^2\Sigma$ bands. The BeF bands do not appear in absorption under similar conditions.

WE have reported[1] qualitative observations on the absorption bands of MgF by using a 21-foot grating. The spectrum described by Walters and Barratt[2] was shown not to belong to this molecule. In absorption, we observe only the $^2\Sigma$, $^2\Sigma$ and $^2\Pi$, $^2\Sigma$ systems known in emission[3, 4] and a new system near λ2275. The latter apparently consists of a single sequence ($v'-v''=0$) of a doublet system, and may be analogous to the system designated[5] as $b^2\Pi$, $a^2\Sigma$ in CaF, SrF and BaF. Fig. 1e is a microphotometer curve of this group. It shows, besides the short sequence of band heads for which wave-lengths have been given,[1] an underlying continuous band with maximum absorption at λ2277.09. As a whole, it closely resembles the $+1$ sequence in the $a\,^2\Pi$, $^2\Sigma$ CaF bands,[6] and is probably to be interpreted similarly. The vibrational constants cannot be determined, because only one sequence is observed.

Measurements of our absorption plates have been made for the longer wave-length systems in order to find (1) approximate rotational constants, (2) the electronic doublet separation in the $^2\Pi$ state, and (3) definite evidence for the isotope effect of Mg. Much greater resolution than that afforded by a 21-foot grating is required to apply the combination principle to individual band lines. Enough of the rotational structure is resolved, however, to obtain some information on points (1) and (2).

$^2\Sigma$, $^2\Sigma$ SYSTEM

Figs. 1c and d shows the 0 sequence as it appears in absorption, and a microphotometer curve of this group. On the absorption spectrogram, measurements were made of 29 resolved lines, which obviously belong to the R branch of the strong 0,0 band. These could be accurately represented by the parabolic equation

$$\nu=37{,}259.49+2.5539\,M+0.01875\,M^2, \quad (1)$$

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[1] F. A. Jenkins and Rafael Grinfeld, Phys. Rev. **43**, 943 (1933).

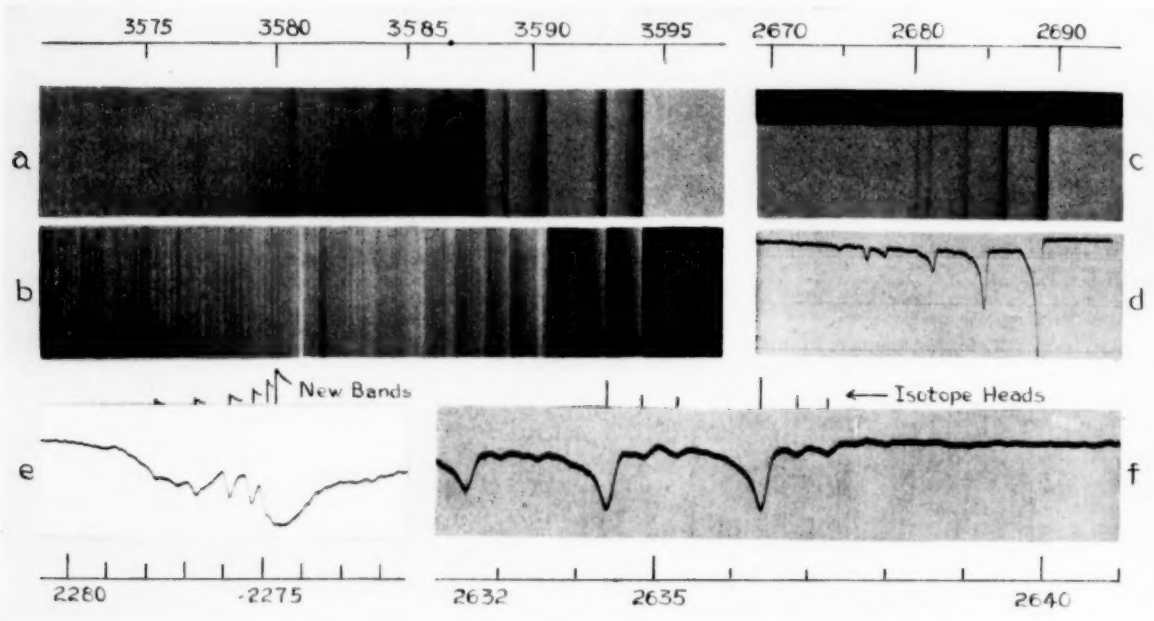
[2] O. H. Walters and S. Barratt, Proc. Roy. Soc. **A118**, 134 (1928).

[3] R. C. Johnson, Proc. Roy. Soc. **A122**, 199 (1929).

[4] W. Jevons, Proc. Roy. Soc. **A122**, 223 (1929).

[5] F. A. Jenkins and A. Harvey, Phys. Rev. **39**, 929 (1932).

[6] A. Harvey, Proc. Roy. Soc. **A133**, 336 (1931).



FIGS. 1. a and b. Absorption and emission, respectively, of the sequence $v'-v''=0$ in the $^2\Pi$, $^2\Sigma$ bands of MgF. Second order of the 21-foot grating. c. Absorption of the 0 sequence of $^2\Sigma$, $^2\Sigma$ bands, first order. d. Microphotometer curve of c on a slightly enlarged scale. e. Microphotometer curve of the new MgF bands on a first order plate. Positions of sharp heads are marked above. f. Microphotometer curve of the +1 sequence of $^2\Sigma$, $^2\Sigma$ bands, first order. The two subsidiary heads for each band are due to Mg ^{25}F and Mg ^{26}F, in the order of increasing wave-length.

TABLE I. *Lines of the R branch of the* $^2\Sigma$, $^2\Sigma$ *0,0 band.*

M	(obs.)	(calc.)	$o-c$	M	(obs.)	(calc.)	$o-c$
−14	37,227.38	37,227.41	−0.03	1	37,262.25	37,262.06	0.19
−13	229.34	229.45	−0.11	2	264.60	264.67	−0.07
−12	231.49	231.54	−0.09	3	267.32	267.32	0.00
−11	233.78	233.66	0.12	4	270.09	270.00	0.09
−10	235.86	235.82	0.04	5	272.57	272.72	−0.15
−9	238.03	238.02	0.01	6	275.29	275.49	−0.20
−8	240.24	240.26	−0.02	7	278.40	278.28	0.12
−7	242.73	242.53	0.20	8	281.24	281.12	0.12
−6	244.75	244.84	−0.09	9	283.69	283.99	−0.30
−5	246.97	247.19	−0.22	10	286.56	286.90	−0.34
−4	249.57	249.19	−0.38	11	289.89	289.85	0.04
−3	252.12	251.99	0.13	12	292.91	292.83	0.08
−2	254.44	254.45	−0.01	13	295.99	295.86	0.13
−1	257.02	256.95	0.07	14	299.06	298.92	0.14
0	259.63	259.49	0.14	—	—	—	—

which was found by least squares. The observed frequencies are compared with their calculated values in Table I. If the band has the structure assumed, this series should converge at the observed frequency of the 0,0 head. We find, from Eq. (1)

$$\nu_h \text{ (calc.)} = 37{,}172.52, \quad \nu_h \text{ (obs.)} = 37{,}172.69,$$

a very satisfactory agreement.

The 0,0 band shows, in absorption, a distinct minimum of intensity not far from the head. This we take to represent the band origin. We have measured its position both visually and on microphotometer curves, and find as a mean value $\nu_0 = 37{,}187.4 \pm 0.4$. This corresponds in Eq. (1) to the practically integral value $M = -39.96$, indicating that the true rotational quantum number for the R branch $K'' = M + 40$.

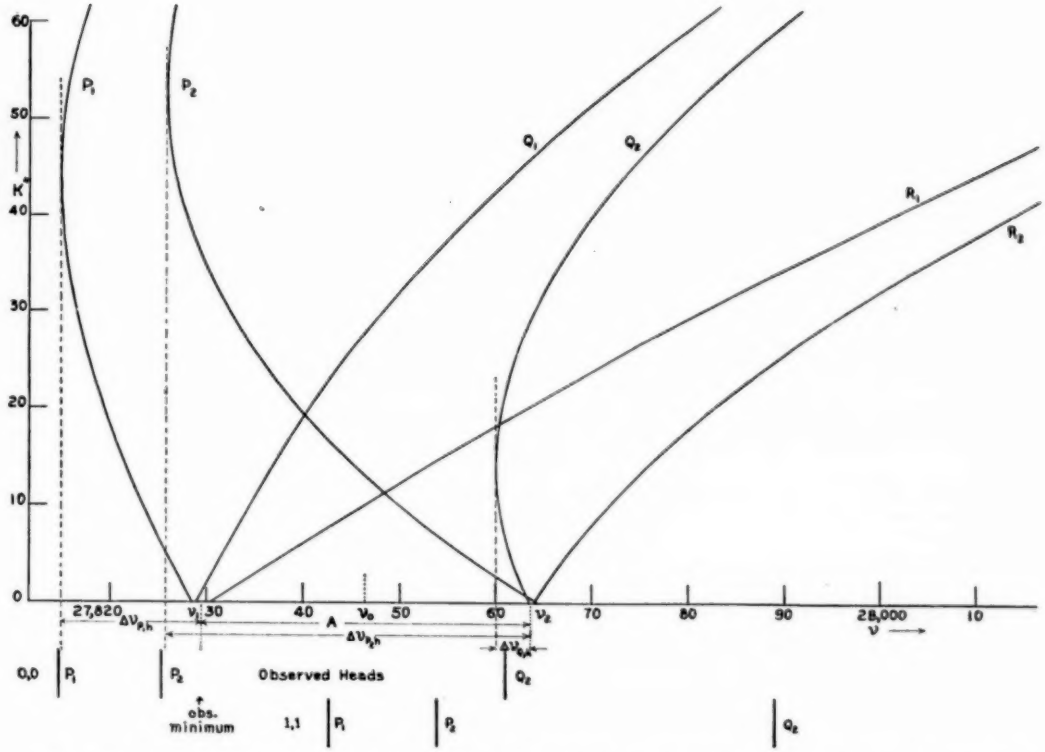


FIG. 2. Fortrat diagram of the 0,0 band of the $^2\Pi$, $^2\Sigma$ system, calculated by Eqs. (4), with the constants of Table II. The measured positions of the heads of the 0,0 and 1,1 bands are indicated below, as well as the position of minimum absorption, corresponding to one of the origins.

In $^2\Sigma$, $^2\Sigma$ bands the spacing of adjacent lines at the origin is[7] $B_0'+B_0''$, and from Eq. (1) we obtain $B_0'+B_0''=1.055\pm0.014$. The coefficient of M^2 in Eq. (1) gives $B_0'-B_0''=0.01875$ fairly accurately. Hence we have

$$B_0'=0.537\ \text{cm}^{-1}, \qquad I_0''=53.4\times10^{-40}\ \text{g cm}^2,$$
$$B_0''=0.518\pm0.010, \qquad r_0''=1.75\times10^{-8}\ \text{cm}.$$

This value of r_0'' fits well with the known values 1.36×10^{-8} for BeF[8] and 2.0×10^{-8} for CaF.[6]

$^2\Pi$, $^2\Sigma$ System

Here the structure of the bands is more complex, as is evident in Figs. 1a and b. Hence measurements of the resolved lines cannot be as easily interpreted as in the simple $^2\Sigma$, $^2\Sigma$ bands. However, microphotometer curves of the absorption spectrum show two features which are of use. First, there is a distinct minimum at $\nu=27{,}829.4$, slightly on the long wave-length side of the second head of the 0,0 band. We interpret the three heads in each band as due to P_1, P_2 and Q_2 branches,[7] in the order of increasing frequency, and this minimum as the origin ν_1 of the low frequency sub-band, as indicated in Fig. 2. This sub-band is the $^2\Pi_{3/2}$, $^2\Sigma$ component, if the $^2\Pi$ state is inverted, as in BeF.[9] Second, a fairly regular series of lines appears in the tail of the sequence, which probably represents the Q branches of the 0,0 band. The two branches Q_1 and Q_2 must draw rapidly together, and become practically fused into one for high J values.

The position of the origin ν_2 of the $^2\Pi_{1/2}$, $^2\Sigma$ sub-band should lie very close to the Q_2 head, since the head of this branch occurs at a low quantum number. The measured frequency of this head is $\nu_{Q_2 h}=27{,}861.02\sim\nu_2$. The mean of this and of the value of ν_1 found above gives the preliminary value $\nu_0=27{,}845.2$ as the origin of the whole 0,0 band. The lines of the assumed Q branch (mean of Q_1 and Q_2) should be given by $\nu=\nu_0+CM^2$, and their spacing by $\Delta\nu=2CM$.

[7] R. S. Mulliken, Rev. Mod. Phys. **3**, 89 (1931).
[8] F. A. Jenkins, Phys. Rev. **35**, 315 (1930).
[9] R. S. Mulliken, Phys. Rev. **38**, 836 (1931).

The spacing of the regular lines observed near the tail is 2.09 at $\nu = 27{,}952$. Using the above expressions for ν and $\Delta\nu$, we obtain as a preliminary value $C = 0.0103$. From this, $B' = (B_1 + B_2)/2 = B'' + C = 0.518 + C = 0.528$, since the value of B'' must be the same for this system as that found above for the $^2\Sigma$, $^2\Sigma$ system. A better value for ν_2 may now be obtained by correcting for the distance from the head to the origin $\Delta\nu_{Q_2 h} = B_2^2/4C_2$. C_2 may be found from the relation[7] $C_2 = C - 2B''^2/A$, where A is the preliminary value of the electronic separation, -31.6 cm^{-1}. This yields $C_2 = 0.028$, and $\Delta\nu_{Q_2 h} = 2.7$ cm^{-1}. We therefore have, for the origins of the 0,0 band:

$$\nu_1 = 27{,}829.4, \qquad \nu_2 = 27{,}863.7, \qquad A = -34.3.$$

A more reliable value of C may now be obtained by applying the known formulas[7] for the separation of the P heads from the origins in this type of band, namely

$$\Delta\nu_{P_1 h} = B_1^2/4C_1, \tag{2}$$

$$\Delta\nu_{P_2 h} = (B_2 + 2B'')^2/4C_2. \tag{3}$$

Using the measured values of P_{1h} and P_{2h} given in Table II, we obtain from Eq. (2) $C_1 = 0.0044$ and from Eq. (3) $C_2 = 0.0170$. The former is somewhat too large, because of the rotational distortion, and the latter too small for the same reason. But the mean value, $C = 0.0107$ should be accurate, and agrees well with the above preliminary value, 0.0103.

As a check on the value of the constants we have obtained, we have used them in the general Hill and Van Vleck term-formula for $^2\Pi$ states, and computed the frequencies of the band heads. With the equations

$$\begin{aligned} \nu &= \nu_0 + F' - F'' \\ F'_{2,\,1} &= B'\left[(J+\tfrac{1}{2})^2 - \Lambda^2 \pm \{(J+\tfrac{1}{2})^2 - (A/B)(1 - A/4B)\}^{\frac{1}{2}}\right] \\ F'' &= B''K(K+1), \end{aligned} \tag{4}$$

with the constants given in Table II, the minimum values of ν for the P_1, P_2 and Q_2 branches can be found. The agreement with the measured values, shown in Table II, could probably be improved by further adjustment of the constants.

It is of interest that the modifications in relative intensity of the heads in passing from emission (arc, $T \sim 4500$°C) to absorption (furnace, $T = 1200$°C) are in exact accord with what we should expect due to the difference in temperature. In particular, the Q_2 heads, which occur at low J values, are greatly enhanced in absorption.

TABLE II. *Adopted constants and computed positions of the heads of the 0,0 $^2\Pi$, $^2\Sigma$ band.*

$A = -34.3$, $B'' = 0.5180$, $B' = 0.5287$, $\nu_0 = 27{,}846.5$.

	ν_h(calc.)	ν_h(obs.)	J_h(calc.)
P_{1h}	27,814.7	27,814.4	$14\frac{1}{2}$
P_{2h}	27,825.7	27,825.2	$43\frac{1}{2}$
Q_{2h}	27,860.0	27,861.0	$52\frac{1}{2}$

Our value of the electronic separation in the $^2\Pi$ state, $A = -34.3$, differs from the previous estimates of 22 and 18.6 given by Mecke[10] and by Johnson,[3] respectively. However, these authors used the separations of certain *heads*, which do not even approximately represent the separations of the band origins where the rotational distortion is as great as in the present case ($A/B = -66.2$). In our work we have assumed a negative sign for A (inverted $^2\Pi$ state) merely by analogy with the corresponding bands in BeF.[9] If the analogy does not hold, and the doublet is normal, an equally good fit could be obtained by using $A = +38.3$ in the Hill and Van Vleck formula.

Isotope Effect

Some evidence for the Mg isotope effect in the $^2\Pi$, $^2\Sigma$ bands has been found by Johnson[3] in the measurements of the emission spectrum by Datta.[11] Magnesium has isotopes 24, 25 and 26 with an abundance ratio 7 : 1 : 1. Johnson identified 8 faint heads associated with the Q_2 heads of the 1,0 sequence, which he attributed to Mg ^{25}F. Since the observed shifts varied between 4.31 and 6.99 cm^{-1} (theoretical value ~ 6.4) and since no corresponding heads of Mg ^{26}F were found the evidence is not satisfactory.

The absorption spectrum is more suitable for the detection of faint isotopes. Since we first reported[1] two isotope heads associated with the 1,0 band of the $^2\Sigma$, $^2\Sigma$ system, plates showing stronger absorption have been taken, with the absorption tube at 1240°C. Fig. 1f shows a

[10] R. Mecke, Zeits. f. Physik **42**, 390 (1927).
[11] S. Datta, Proc. Roy. Soc. **A99**, 436 (1921).

TABLE III. *Isotope shifts in the* $^2\Sigma$, $^2\Sigma$ *system.*

Band	Mg ^{24}F	Mg ^{25}F	Mg ^{26}F	$\Delta\nu$ (24,25) obs.	$\Delta\nu$ (24,25) calc.	$\Delta\nu$ (24,26) obs.	$\Delta\nu$ (24,26) calc.
1,0	37,918.9	37,912.5	37,906.8	6.4	6.92	12.1	13.35
2,1	947.8	940.8	934.9	7.0	7.18	12.9	13.85
3,2	973.9	(967.6)	960.5	(6.3)	7.41	13.4	14.30

microphotometer curve of the +1 sequence on these plates, and the isotopic bands due to Mg ^{25}F and Mg ^{26}F are apparent, not only for the 1,0 band, but also for 2,1 and 3,2. Measurements on these curves of the isotope shifts are given in Table III. The theoretical shifts were calculated by the approximate relation $\nu_v{}^i-\nu_v=(\rho-1)\nu_v$, with $\rho=1.00888$ for Mg ^{25}F and 1.01714 for Mg ^{26}F. The agreement is rather poor, because of the faintness of the heads and the neglect of the rotational shift.

In conclusion, it seems of value to report attempts to observe the BeF bands in absorption. This would be very desirable in order to verify the reported existence of the isotope Be^8. However, by using a procedure similar to that for MgF, that is, by heating pure BeF_2 with metallic Be and Mg, no absorption was obtained up to temperatures of 1200°C.

Electromagnetic Waves of 1.1 cm Wave-Length and the Absorption Spectrum of Ammonia

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A spectrometer, with a grating of the echelette type, was used in mapping the absorption spectrum of ammonia gas in the region from 1 to 4 cm wave-length. To accomplish this, it became necessary to produce electromagnetic waves of shorter wave-length than had been produced before by vacuum tubes. Oscillators of the magnetron type were used as the source of the continuous short wave radiation. The center of the absorption band had been predicted to be at 1.5 cm wave-length. The maximum absorption was found to occur at 1.25 cm. This shift is explained by the unsymmetrical nature of the line. The constants in a theoretical formula for the absorption coefficient have been determined. These constants involve an effective collision diameter of the molecule, and lead to the value $\sigma=8.8\times10^{-8}$ cm. The total intensity of the line was found to be 10×10^{7} cm^{-1} sec.$^{-1}$.

INTRODUCTION

THE ammonia molecule has the form of a regular pyramid with the nitrogen atom at the apex. The problem of the vibrational energy levels of the molecule has been discussed by Dennison and Hardy[1] and by Dennison and Uhlenbeck.[2] It has been shown that the energy levels are split into pairs because there exist two equivalent positions of equilibrium for the nitrogen atom. The energy difference between the two lowest levels gives rise to a doubling of the far infrared absorption lines of ammonia, which has been observed by Wright and Randall.[3] From their measurements this energy difference was found to be 0.67 cm^{-1}. The theory also predicts that ammonia gas should strongly absorb radiation of the frequency, $\nu=0.67$ cm^{-1}, corresponding to a wave-length of 1.5 cm. It is the experimental measurement of this absorption that is reported in this paper.

EXPERIMENTAL

The radiation used consisted of short electric waves produced by a magnetostatic oscillator. The wave-length was measured by a spectrometer with two brass mirrors three feet in diameter and an echelette grating of corresponding size. The ammonia gas was contained in a rubberized cloth cell at about atmospheric pressure.

The term magnetostatic oscillator was coined at the laboratory of the Westinghouse Electric and Manufacturing Company and is applied to electronic oscillators using a vacuum tube having a hot cathode and one or more anodes in combination with a constant magnetic field. The frequency depends primarily upon the time of flight of the electrons between the cathode and the anode. Zacek[4] first discovered this type of electronic oscillations. The reference given below is to some comments which he made on the subject some time after his original paper was published. Later Yagi[5] and Okabe[6, 7] built tubes which produced stable oscillations whose wave-lengths were, in the extreme case, as short as 3.16 cm. They obtained much greater output by splitting the anode into semi-cylinders, and connecting the two halves to form an oscillatory circuit. Kilgore[8] discovered the importance of inclining the magnetic field with respect to the cathode, in order to obtain maximum output. He also gave a thorough discussion of the characteristics of a tube operating at a wave-length of about 42 cm. As a result of the improvements in design made by Kilgore, tubes were built at the Westinghouse laboratories capable of giving considerable power at a wave-length of 9 cm. The

[1] Dennison and Hardy, Phys. Rev. **39**, 938 (1932).
[2] Dennison and Uhlenbeck, Phys. Rev. **41**, 313 (1932).
[3] Wright and Randall, Phys. Rev. **44**, 391 (1933).
[4] Zacek, Zeits. f. Hochfrequenztechnik **32**, 172 (1928).
[5] Yagi, Proc. I. R. E. **16**, 715 (1928).
[6] Okabe, Proc. I. R. E. **17**, 652 (1929).
[7] Okabe, Proc. I. R. E. **18**, 1748 (1930).
[8] Kilgore, Proc. I. R. E. **20**, 1741 (1932).

success of these tubes encouraged the authors to attempt to reduce the wave-length sufficiently to study the absorption by ammonia gas at 1.5 cm. The shorter waves were obtained by building tubes of very small dimensions, following the design of some of the 9 cm tubes kindly loaned to us by the Westinghouse laboratories. The anodes were of graphite, and the Lecher system connected between the anode segments was contained within the tube envelope, a type of construction which Kilgore had demonstrated to be possible. As the dimensions of the tube were decreased the applied magnetic field was increased according to the approximate relation, $\lambda H = 13{,}000$, given by Okabe,[6] in which λ is the wave-length in cm and H is the field intensity in gauss.

One of the smaller tubes, which had an anode radius of 0.027 cm and a Lecher system about 4 mm long, produced waves 1.13 cm long, with 870 volts on the anode and in a magnetic field of 11,000 gauss. The energy at this wave-length was sufficient to produce a full scale deflection of the receiving galvanometer. The characteristics of these tubes were similar to those which produced the longer wave-lengths. Each tube could be made to operate at any frequency within a small range covering about 30 percent of the mean frequency. The tuning was accomplished by varying the magnetic field and by adjusting the plate voltage, the filament current, and the angle between the filament and the field. Four tubes were used in making absorption measurements between wave-lengths of 1.06 cm and 3.8 cm.

The arrangement of the spectrometer[9] is shown in Fig. 1. The oscillator tube was placed between the pole pieces, PP, of a strong electromagnet, and at the focus of the parabolic mirror M_1. A parallel beam of the short wave radiation fell upon the grating G, which has 18 elements and a constant of 7.49 cm. The diffracted beam was focussed on a rather large slit in the metal shield S, by a second parabolic mirror M_2. An iron pyrite-phosphor-bronze crystal detector was placed inside the shield just behind the opening and was connected to a Leeds and Northrup galvanometer. A metal reflector behind the detector could be adjusted along the axis of the mirror for greater sensitivity. This arrangement served as a suitable receiver for the radiation. The elements of the grating were automatically adjusted to remain perpendicular to the bisector of the angle θ as the grating was turned, thereby giving maximum energy at any position of the grating. The reflectors R were arranged to prevent the radiation reflected from the walls of the room from reaching the detector. When they were properly placed, no radiation fell upon the detector except that which came directly from the grating. The alignment of the spectrometer was accomplished by focussing the image of the filament of the oscillator tube upon the slit in the receiver shield.

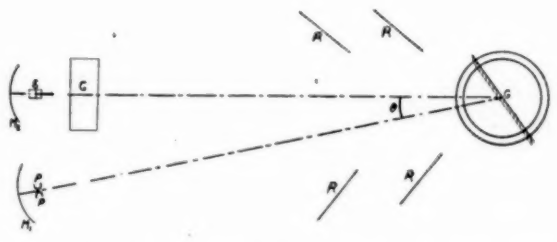

FIG. 1. Diagram of the spectrometer designed for wave-lengths below 10 cm.

The cell C was placed in front of the receiving mirror. The cell was made of rubberized cloth cemented at the seams and supported on two wooden frames, which allowed it to collapse so that the air could be removed and pure ammonia introduced into the cell. The cross section of the cell was 36×45 inches and the length 16 inches. There seemed to be a slight diffusion of the ammonia through the rubberized cloth. To insure that there was the same amount of ammonia in the cell at all times, it was operated at a pressure of about 3 mm of oil. That the air did not diffuse into the cell was indicated by the fact that the cell would hold air at a slight pressure for hours, whereas, when the ammonia diffused outward, the cell would collapse. That the absorption readings could be repeated after the lapse of considerable time was also taken as evidence that the gas remained pure.

The procedure involved in obtaining the absorption curve was as follows. The adjustments were made on the tube such that small variations in plate current, caused by line voltage fluctuations, produced the least effect upon the intensity. The wave-length was determined in

[9] Cleeton and Williams, Phys. Rev. **44**, 421 (1933).

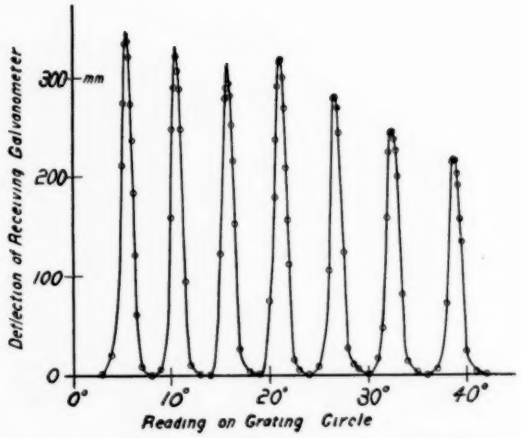

FIG. 2. Seven orders at λ1.33 cm as given by the echelette grating.

three orders by setting the grating at about six positions near the place of maximum energy and finding the peaks of the curve of the intensity plotted against the grating angle. Fig. 2 shows the complete curve for seven orders at a wave-length of 1.33 cm. The inaccuracy of the wave-length as measured by this method is less than one percent. The orders to be used were selected as high as possible, with these two limitations: that the angle between the grating and the axis of the mirror be sufficiently large to permit the full utilization of the beam, and that good definition be maintained. The absorption was determined for the average of five or more pairs of readings, taken with the cell in the path of the beam and then out of the path. After the absorption readings had been taken the ammonia was pumped out, the cell washed several times with air, then filled with air, and the same procedure followed for the same range of wave-lengths. The ammonia absorption data were then corrected for the absorption of the cell.

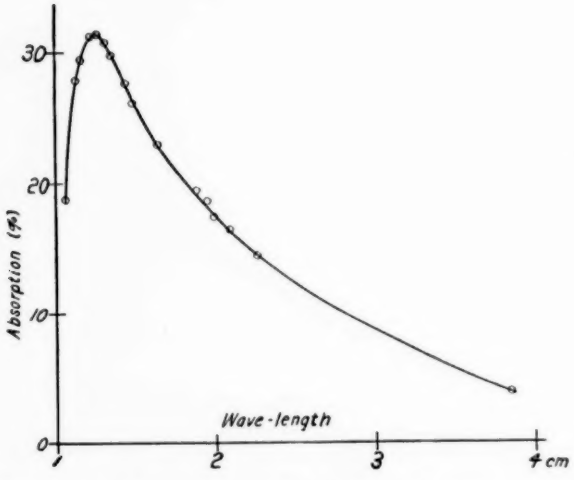

FIG. 3. The absorption band of NH_3 plotted on a wave-length scale.

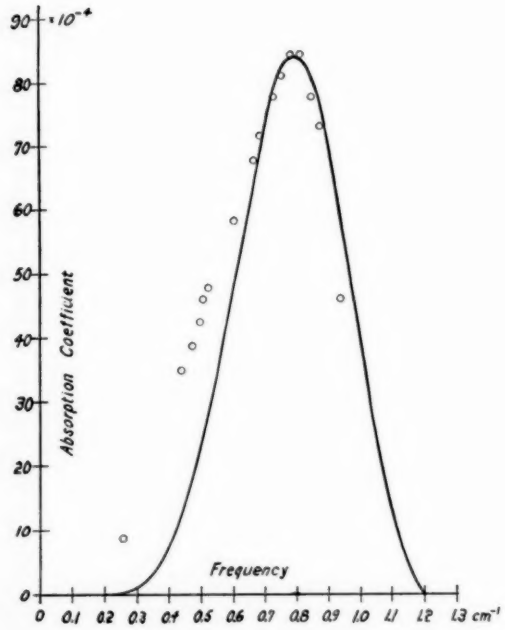

FIG. 4. The theoretical absorption curve for NH_3 and the observed experimental points plotted on a frequency scale.

RESULTS

The experimental absorption curve of the ammonia line after corrections have been made for the absorption of the cell is shown in Fig. 3. The maximum absorption of 31.5 percent occurs at a wave-length of 1.25 cm ($\nu = 0.8$ cm^{-1}).

Dennison[10] has shown that the shape of an infrared absorption line depends mainly on the limitation of the length of the wave trains which may be absorbed by the molecules. This leads to the approximate theoretical formula for the absorption coefficient at any frequency

$$\alpha_\nu = K\nu^2\left[\frac{\sin(\nu_0-\nu)\pi t}{\nu_0-\nu}+\frac{\sin(\nu_0+\nu)\pi t}{\nu_0+\nu}\right]^2$$

where ν_0 is the proper frequency of the molecule, t the time between collisions, and K a constant. This equation assumes that the time during which

[10] Dennison, Phys. Rev. **31**, 503 (1928).

a collision takes place is very short compared with the time t between collisions. We will take t to be the mean time between collisions. This assumption involves an approximation which may well explain the divergence between the experimental points and the theoretical curve. A more nearly exact theoretical relation might be obtained by using a statistical average over the time between collisions. Fig. 4 shows this theoretical curve, where K and t were adjusted to give the best appearing fit to the experimental points, and the value of ν_0 was taken as 0.67 cm^{-1} from the results of Wright and Randall at 100μ. The absorption coefficient α_ν, plotted as ordinate, is related to the percentage absorption under the special conditions by the equation

$$\text{percent absorption} = 1 - e^{-\alpha_\nu l}$$

in which l is the path length under the conditions of pressure and temperature at which the measurements were made. The kinetic theory of gases furnishes the relation among t, n, the number of molecules per unit volume, $\bar{c}$ the mean value of the velocity of each molecule and σ the effective diameter of the molecule,

$$\bar{t} = 1/(1.41\pi\sigma^2 n\bar{c}).$$

The value of t selected for the theoretical curve was 0.57×10^{-10} sec. This gives a value of 8.8×10^{-8} cm for σ, the effective diameter for this kind of process. This is of the right order of magnitude. It need not agree with the gas kinetic cross section since our σ is the minimum distance to which a molecule may approach another molecule without interrupting its wave train. The total intensity of absorption is defined by $\alpha = \int \alpha_\nu d\nu$. The value of α as determined from the experimental curve is 10×10^7 $sec.^{-1}$ cm^{-1}.

We wish to thank Dr. I. E. Mouromseff, Head of the Physics Division of the Westinghouse laboratories for loaning us some 9-cm tubes, and Dr. H. N. Kozanowski of the same laboratory for information concerning the technique of building the tubes. Also we are grateful to Professor D. M. Dennison for his advice and assistance in preparing the theoretical part of this paper.

Infrared Absorption Spectra of Phosphine

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The 10μ absorption region of PH_3 has been resolved into three bands, a pair with centers at 1121 cm^{-1} and 992 cm^{-1} in which the line intervals are approximately 11 cm^{-1}, and a single band at 990 cm^{-1} with line intervals of approximately 8.5 cm^{-1}. The 4.3μ band has intervals of approximately 8.5 cm^{-1} and its center is at 2327 cm^{-1}. All apparently have zero branches, as do the two higher frequency bands at 3428 cm^{-1} and 4541 cm^{-1}. No absorption was observed between 16μ and 22μ. The two parallel type bands may be identified by comparison with the pure rotation spectrum observed by Randall and Wright. They are the ones at 990 cm^{-1} and 2327 cm^{-1}. The doubling observed in the corresponding bands of NH_3 does not appear here; hence the phosphorus atom cannot pass readily through the plane determined by the three hydrogen atoms. The perpendicular type bands have intense central maxima, and do not show an intensification of every third line as in NH_3. This indicates that PH_3 behaves like a spherical top. The moment of inertia about the axis of symmetry and the interatomic distances cannot be determined precisely until information is available concerning the coupling of rotation and vibration.

THIS study of the infrared bands of phosphine was undertaken not only for the information which it might yield regarding PH_3 but also in the hope that it might throw some further light upon the problem of ammonia. It is quite certain that the form of the NH_3 molecule is a low pyramid with the three hydrogen atoms located at the corners of its equilateral triangular base, thus constituting one of the simplest axially symmetrical systems. The spectrum is extensive and fairly well known, but its correlation with the intramolecular motions is not yet complete. Should PH_3 prove to be similar mechanically, then the points of correspondence and of difference in the two spectra might assist in the problem of interpretation for both.

The spectrum of phosphine has been mapped very accurately and completely by Robertson and Fox,[1] but with a resolution not quite adequate to show conclusively the character of the different bands. The most striking feature reported by them is a pair of intense bands near 10μ with zero branches separated by about 130 wave numbers. There is a temptation to correlate this group at once with the well-known double band of NH_3 at 10μ where the splitting results from the two minima in the potential energy function on opposite sides of the base plane. This would suggest that the corresponding minima for PH_3 must be separated by a very low potential barrier, since the apparent splitting is large, and hence that the phosphorus atom lies very near to the plane of the hydrogen atoms. A more intimate study of these bands however shows that this conclusion is not justified, and that the correspondence with the NH_3 band is entirely superficial.

EXPERIMENTAL

A supply of PH_3 was generated by dropping 50 percent KOH solution upon crystals of phosphonium iodide contained in a flask which had been initially evacuated. The evolving gas was passed through P_2O_5 and CaOH, and then frozen in a trap immersed in liquid air. A first fraction was discarded. After a sufficient amount had collected the generating flask was cut off and part of the phosphine vaporized into a storage reservoir with a mercury manometer. The remainder was also discarded. In withdrawing a sample for use a sufficient quantity was always frozen out in the trap, and then allowed to vaporize to the desired pressure in the previously evacuated absorption cell. Then, after the stopcocks on the cell had been closed, the gas remaining in the connecting tubes was frozen once more and returned from the trap to the reservoir. With this procedure very little trouble was experienced as long as the initial vacua were sufficiently good.

[1] Robertson and Fox, Proc. Roy. Soc. **A120**, 161 (1928).

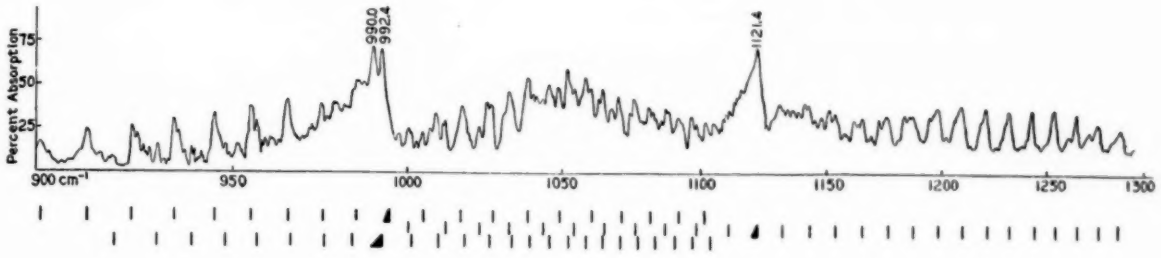

FIG. 1. The 10μ bands of PH_3.

A trace of air is instantly indicated by a bright flame which leaves a solid deposit upon the cell windows and greatly reduces their transmission. Two absorption cells were used, one 11 cm long with windows of NaCl, and one 2.5 cm long with KBr windows. Two spectrometers were also available, one being completely enclosed so that CO_2 could be excluded from the optical path. Unfortunately this one had a less effective optical system and gave only moderate resolution.

The structure observed in the 10μ region is shown in Fig. 1, the curve representing absorption of 11 cm of gas at about 30 cm pressure. The second order of a 1200 line grating was used. It is clear that the pattern does not consist simply of a pair of parallel type bands. The two strong maxima at 992 cm^{-1} and 1121 cm^{-1} suggest zero branches, but the former is accompanied by a close neighbor at 990 cm^{-1}. Upon a reduction of the gas pressure to 2 cm all the weaker portions of the structure vanish, but these three peaks persist with approximately equal intensities. On both high and low frequency sides of the region lesser absorption maxima occur at almost equally spaced intervals. They do not have the appearance of simple rotation lines, however. Near the center many poorly resolved maxima are seen. An attempt has been made to assign all of the strong lines to three band series, although this may not be entirely convincing. The bands are indicated diagrammatically below the curves. The two associated with the centers at 992 cm^{-1} and 1121 cm^{-1} have approximately equal spacing of about 11 cm^{-1}, while the intervals in the third band are only about 8.5 cm^{-1}, showing that it must be of quite a different character. The positions of these lines and their wave-number differences are given in Table I. It would seem as if the irregularities are rather greater than would have been expected even when account is taken of the effects of overlapping of the three bands, and of upper stage components due to absorption by excited molecules.

TABLE I. *The 10μ bands.*

Line No.	ν	$\Delta\nu$	ν	$\Delta\nu$	ν	$\Delta\nu$
−8	919.5	9.6	913.0	10.5	1030.8	11.5
−7	929.1	10.2	923.5	10.5	1042.4	10.5
−6	939.3	8.9	934.0	10.8	1052.8	11.6
−5	948.2	8.2	944.8	10.2	1064.4	11.4
−4	956.4	9.2	955.0	9.7	1075.8	11.2
−3	965.6	8.9	964.7	9.6	1087.0	10.9
−2	974.5	15.5	974.3	18.1	1097.9	10.6
−1					1109.5	11.9
0	990.0		992.4		1121.4	
+1	1000.7	10.7	1005.0	12.6	1130.5	10.1
+2	1009.1	8.5	1015.8	10.8	1142.2	11.7
+3	1017.6	8.5	1026.5	10.7	1153.3	11.1
+4	1025.3	7.7	1036.9	10.4	1164.2	10.9
+5	1032.4	7.1	1048.6	11.7	1175.2	11.0
+6	1038.9	6.5	1060.0	11.4	1186.7	11.5
+7	1045.6	6.7	1070.0	10.0	1198.1	11.4
+8	1051.6	6.0	1080.2	10.4	1209.0	10.9
+9	1058.0	6.4	1090.8	10.6	1220.3	11.3
+10	1064.4	6.4	1101.3	10.5	1231.4	11.1

The question must be raised whether or not these three bands are all fundamentals. In a symmetrical molecule the harmonic of a perpendicular type band is double, and consists of one perpendicular and one parallel component, slightly separated. This suggests the situation at 990 and 992 cm^{-1}. We have taken observations through the region from 16μ to 22μ with the short cell and found no positive evidence of absorption, although a band of the required intensity could hardly have escaped detection. Dr. Wright has also been kind enough to make a search near 20μ with the large grating, using reststrahlen for isolating the region and his results are also negative. The absence of low frequency bands leaves no alternative to the assignment of three fundamental frequencies at 990, 992 and 1121 cm^{-1}.

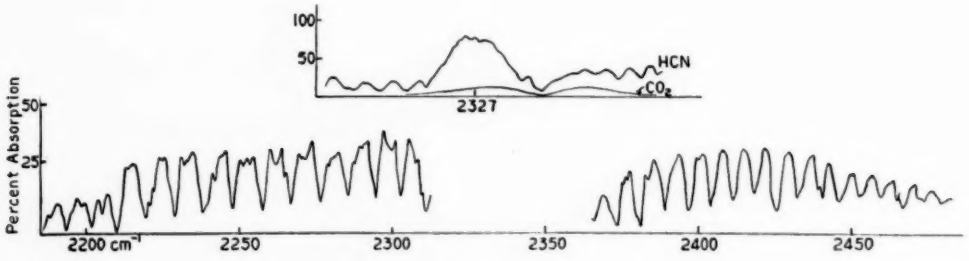

FIG. 2. The band at 4.3μ.

A second region of intense absorption is found at 4.3μ. Robertson and Fox have correlated this with the 3μ band (ν_1) of NH_3. The middle portion of this band is almost completely cut out by the very strong atmospheric CO_2 absorption, but the two wings are shown in the lower curve of Fig. 2 as mapped with a 4800 line grating and a slit including about one wave number. The upper curve of Fig. 2 shows the same band as measured with the enclosed spectrometer containing air almost free of CO_2. The background curve taken with the absorption cell removed shows only a slight indication of the CO_2 doublet. The center of the rather broad zero branch lies at 2327 cm^{-1} and the spacing of the rotation lines is about 8.5 cm^{-1}. The structure suggests a parallel type band as expected, with some complications perhaps from upper stage transitions. The line positions and intervals are shown in Table II.

Two weaker bands have been observed with low resolution, one at 2.9μ (3428 cm^{-1}) and the other at 2.2μ (4541 cm^{-1}). The contours look much alike and both show intense zero branches.

TABLE II. *The 4μ band.*

Line No.	ν	$\Delta\nu$	Line No.	ν	$\Delta\nu$
0	2327		−2	2307	
+5	2370		−3	2298	9
+6	2377	7	−4	2290	8
+7	2386	9	−5	2281	9
+8	2394	8	−6	2272	9
+9	2401	7	−7	2262	10
+10	2408	7	−8	2253	9
+11	2415	7	−9	2244	9
+12	2422	7	−10	2233	11
+13	2430	8	−11	2224	9
+14	2437	7	−12	2215	9
+15	2443	6			
+16	2450	7			

DISCUSSION

If it be assumed that the PH_3 molecule has the form of a symmetrical pyramid, there should be two fundamental bands of the perpendicular type and two of the parallel type, the former with many zero branches, and the latter with one or at most two. The line spacing must be the same for both parallel type bands, and the same as that of the pure rotation spectrum. It seems reasonable to interpret the band at 2327 cm^{-1} as ν_1, the higher frequency parallel vibration; ν_3 must then be the band at 990 cm^{-1}. This assignment is confirmed by the observations of Randall and Wright[2] upon the pure rotation spectrum, which were made while this study was in progress upon a sample of our gas. They found an average interval of 8.7 cm^{-1} between four successive lines, and no indication of the doubling which is characteristic of NH_3. It is necessary to conclude that the bands at 992 and 1121 cm^{-1} must represent the independent perpendicular vibrations ν_2 and ν_4. The higher frequency bands seem to involve combinations of ν_1 with ν_4, neither of the other frequencies appearing as differences.

The spacing in the parallel bands provides a measure of the moment of inertia about an axis normal to the symmetry axis which is in good agreement with the value 6.22×10^{-40} computed by Randall and Wright. The absence of observable doubling in these bands, and in the pure rotation lines, shows that the pyramid cannot be extremely low as at first supposed. A precise determination of the height of the pyramid, and of the moment of inertia with respect to the axis of symmetry, will be possible only when more in-

[2] Randall and Wright, Phys. Rev. **44**, 391 (1933).

formation is available with regard to the coupling between rotation and the perpendicular vibrations.

Of the six bands which have been observed, every one appears to have an intense zero branch. None shows the form usually characteristic of a perpendicular band attributable to an axially symmetrical molecule, and there is no indication of an accentuation of intensity in every third line, such as appears in perpendicular bands of NH_3 and the methyl halides, indicative of the equilateral triangular arrangement of the hydrogen atoms. These facts may be harmonized by supposing that PH_3 behaves like a *spherical* top, i.e., as if its three moments of inertia were all equal. This would result in a superposition of the various components of any perpendicular band, effectively masking variations in their relative intensities, and producing a single sharp central maximum. If rigid coupling between rotation and vibration be assumed, so that the apparent equality of moments of inertia becomes real, then the molecular dimensions could be computed at once. The distances between hydrogen atoms would be 1.9×10^{-8} cm, from the phosphorus atom to any hydrogen atom 1.5×10^{-8} cm, and the height of the pyramid would be 1.0×10^{-8} cm. While these values are not unreasonable it should be emphasized that the apparent moment of inertia with respect to the figure axis may be appreciably different from the actual one, and that these dimensions cannot be considered as final. The difference of spacing in the parallel and perpendicular type bands is an obvious indication that vibration and rotation are not rigidly coupled.

Raman spectra, if available, might be expected to contribute important information regarding PH_3.

The Disintegration of Deutons by High Speed Protons and the Instability of the Deuton

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To obtain evidence of the instability of the deuton and the low mass of the neutron, even more direct than those cited in previous communications, we have compared the effect of proton bombardment on pairs of targets containing ordinary and heavy hydrogen. In all cases an excess of long range protons was obtained from the targets containing H^2. These are attributed to the disintegration of the deuton by the bombarding protons. The range of the protons so obtained agrees with that calculated from the masses and energies involved, on the assumption that the mass of the neutron is in the neighborhood of unity.

LAST spring,[1] when we first bombarded various targets with high velocity deutons, there appeared, in addition to particles which could be attributed to specific atomic disintegrations, a group of protons from all targets for which there seemed to be no satisfactory explanation except that they resulted from the decomposition of the deuton itself. This essential instability of the deuton could be reconciled with known atomic weights only by assigning to the neutron a much lower mass than had been assigned to it. Chadwick had obtained the value 1.0067 and Curie and Joliot[2] have evidence of the still higher value 1.011, while our interpretation of the results of bombardment by deutons required a value for the atomic weight of the neutron much closer to unity. Our assumption implied that together with the characteristic protons (of about 18 cm range) an equal number of neutrons would be emitted. The predicted stream of neutrons, greater from those targets from which more protons are emitted, and approximately equal in number to the protons, has been found.[3]

There remained the possibility that some contaminant common to all of the targets might be responsible for the observed effects, although a series of measurements with several sets of carefully cleaned targets showed the same phenomena. It therefore became desirable to devise an experiment which would, with the least possible ambiguity, demonstrate the instability of the deuton and the low mass of the neutron. Instead of using the deutons as missiles it was decided to bombard with protons pairs of targets just alike except that one of each pair would contain ordinary hydrogen and the other heavy hydrogen.

The first pair of targets $Ca(OH^1)_2$ and $Ca(OH^2)_2$ were prepared by mixing pure calcium oxide with ordinary water and with (nearly 100 percent) heavy water. The two pastes were spread, and allowed to dry in vacuum, on fine platinum gauze. Being prepared from the same calcium oxide, and as far as possible in an identical manner, they were mounted symmetrically in the apparatus, where they were to be bombarded with a stream of hydrogen molecule ions H^1H^{1+}. We were at first surprised to find an appreciable emission of long range protons from both targets. This could not be due to the high speed protons and was found to be caused by the extremely small amount of H^2 in ordinary hydrogen, together with some contamination with heavy hydrogen caused by previous experiments. By parallel experiments with other targets it was established that the single deuton H^{2+} was present in the stream of H^1H^{1+} to the extent of one part in twenty or thirty thousand. This complicates but in no way diminishes the significance of the results about to be described.

The emitted protons were registered by a

[1] Lawrence, Livingston and Lewis, Phys. Rev. **44**, 56 (1933).

[2] Chadwick, Proc. Roy. Soc. **A136**, 692 (1932); Curie and Joliot, Comptes Rendus **197**, 237 (1933).

[3] Livingston, Henderson and Lawrence, Phys. Rev. **44**, 781 (1933).

Thyratron counter connected to a linear amplifier and ionization chamber. The ionization chamber having a depth of 4 mm subtended a solid angle of about 1/40 at the target. Different aluminum and mica foils of known stopping power could be placed directly in front of the ionization chamber and the bias voltage on the Thyratron grids could be varied. Thus with a high bias potential only protons near the end of their range were counted. The incident ions had an energy corresponding to 3.0 Mv (million volts) so that the ion H^1H^{1+} in nuclear encounters had the effect of two protons of 1.50 Mv each. But the contaminant H^{2+} actually struck nuclei with the great energy of 3.0 Mv, which accounts for the large effects produced by a small number of these particles.

Fig. 1 shows the number of proton counts per minute for a current of 10^{-7} amp. of ions striking the targets. The abscissas represent the air equivalent of the foils through which the emitted protons had passed before entering the ionization chamber. The plain circles show the results obtained with the target $CaO(H^2)_2$ while the filled circles show the corresponding results with the target $Ca(OH^1)_2$. For comparison similar observations with a lithium target are shown by the crosses. It is certain that neither lithium nor $Ca(OH^1)_2$ would give any of these long range protons under bombardment by a pure proton beam. That in the observed cases the counts were due solely to deutons is shown by the fact that the relative number and the ranges of the emitted protons from various targets produced by this beam corresponded entirely to those obtained with a beam containing a relatively large fraction of deutons. The target $Ca(OH^2)_2$ was the only exception.

In order to examine further the distribution of disintegration protons with respect to range, we raised the bias voltage from 10.5 to 22.5 and again to 27 v, so that only protons near the end of their range could be counted. The results are shown in Fig. 2 where the difference in counts between the two targets is plotted against the stopping power of the foils. The circles and crosses show the observations with bias voltages of 27 and 22.5 respectively. These points show roughly the range distribution of the emitted protons.

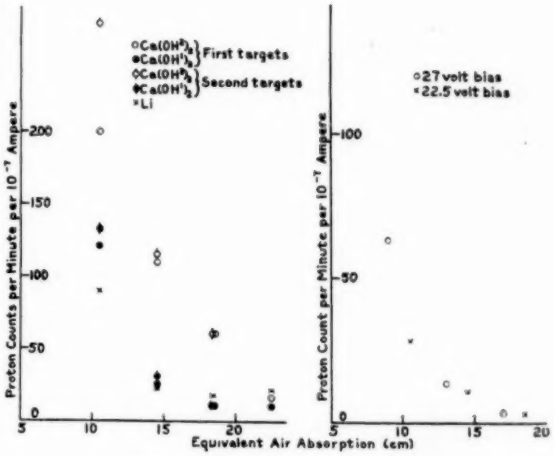

FIG. 1.

FIG. 2.

FIGS. 1 AND 2. Range distribution of protons.

In order to make sure that the difference between the targets $Ca(OH^1)_2$ and $Ca(OH^2)_2$ was not due to some accidental difference in their form or substance, two new targets were prepared from a new and very pure sample of lime. Between the measurements an accident required complete rebuilding of the ionization chamber and some modifications of the amplifier. The result was that the number of disintegrations counted from a given target was diminished by a factor of five. Whether this was due to changes in the geometry of the system or to changes in amplification, equivalent to an increase in bias potential, is being investigated. In the meantime to bring the new results into conformity with the old we have multiplied the numbers of counts in the new experiments by this empirical factor, 5. This has been done in Fig. 1 where the results with the new targets, $Ca(OH^1)_2$ and $Ca(OH^2)_2$, are shown respectively by the same symbols as before, with vertical bars. Here again we see the larger number of long range protons from the target containing heavy hydrogen, which we ascribe to the disintegration of deutons by the proton beam. That this effect is not due to the small contamination of deutons in the beam acting upon the deutons in the target is shown by experiments in which a beam containing a relatively large number of deutons struck the two targets successively. In this case only a slight difference between the two targets was observed, and more refined experiments will be necessary

to show whether there is a specific action of deutons on deutons.

Finally, we attempted to use a pair of targets composed of lithium hydroxide containing the two kinds of hydrogen. Lithium oxide was fused upon two pieces of platinum gauze and then treated with light and heavy water. Unfortunately, the water entered the fused oxide only very superficially and therefore the results were not satisfactory for quantitative interpretation. However, it is highly corroborative of our previous results that in each of three sets of measurements the number of disintegrations counted for the $LiOH^2$ target was greater than that of the $LiOH^1$ target, although the maximum difference was only 60 percent of the number of counts due to the contaminating deutons.

Our receiving apparatus registers particles coming off nearly at right angles to the primary beam. Simple calculations of energy and momentum show that the maximum energy of such particles is equal to one-third of the kinetic energy of the incident particle plus two-thirds of the energy of disintegration. If we take the accepted values 1.0078 and 2.0136 as the masses of H^1 and H^2 and the best value for the neutron obtained from our previous work, namely 1.001, the energy set free in the disintegration of the deuton is 0.0048 mass units or 4.5 Mv. Taking two-thirds of this together with one-third of 1.50 Mv we have 3.5 Mv as the maximum energy of the emitted protons. This corresponds to a maximum range of 18.1 cm (of air, 20°, 760 mm Hg). We see from Figs. 1 and 2 that this is not far from the actual maximum range observed, although further and more careful measurements will be necessary to establish this relation quantitatively.

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On the Theory of the Electron and Positive

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In this paper we develop Dirac's suggestions for the interpretation of his theory of the electron to give a consistent theory of electrons and positives. In Section 1, we discuss the physical interpretation of the theory, the limits which it imposes on the spatiotemporal description of a system and in particular on the localizability of the electron. In Section 2, we set up the corresponding formalism, introducing wave functions to describe the state of the electrons and positives in the system, and constructing operators to represent the energy, charge and current density, etc. It is shown that the theory is Lorentz invariant, and has just that invariance under contact transformations which the physical interpretation requires. The electromagnetic interaction of the electrons and positives is formulated, and certain ambiguities which arise here are discussed. In Section 3, it is shown that in all problems to which the Dirac equation is directly applicable it gives the correct energy levels for an electron, and the correct radiative and collision transition probabilities. In these problems the wave functions are constructed from the solutions of the Dirac equation. In Section 4, we discuss certain problems which have no analogue in the original Dirac theory of the electron, show that a certain part of the energy of an electromagnetic field in general resides in the electrons and positives, and consider the extent to which, in the present state of theory, this can be detected by experiment. For two charges within a Compton wave-length of each other the law of force is not quite Coulomb's law. The deviations though small should in principle be detectable when protons are scattered in hydrogen.

1.

WITH the discovery of the positive electron, one of the most curious and radical predictions of Dirac's theory of the electron has received experimental support. It appears that the positive electron has the properties of the anti-electron of the Dirac theory, and that in some cases, notably in the absorption and internal conversion of gamma-rays, the circumstances of the production of pairs of electrons and positives are just those to be expected from the theory. We want to develop the formalism of the theory in its present form as clearly and consistently as possible, and to show how the formalism is to be interpreted physically, where it justifies earlier methods of calculation, where it leads to new predictions, and where these predictions fail. We shall see that the present theory is limited in very much the same way as the quantum theory of wave fields: here, too, it is possible to develop a consistent and relativistic formalism which within wide limits corresponds to the possibilities of physical observation and gives correct results; but here, too, the legitimate application of the theory is limited to lengths which are large compared to the classical electron radius e^2/mc^2; and when not so limited, leads at once to grave contradictions with experiment.

The formal changes which are required in the theory are simple, and correspond closely to Dirac's most recent suggestion[1] for interpreting the negative kinetic energy states which are falsely predicted by his theory of the electron in its original form. These formal changes may most easily be formulated with the help of the distinction made by Schroedinger,[2] in his attempt to resolve the difficulties with the negative energy states, between odd and even operators, and show how this distinction is properly to be applied to give a consistent and Lorentz invariant theory. This distinction between even and odd operators is essential to the physical interpretation of the theory: for only those dynamical variables of a system which correspond to even operators may be determined without rendering the number of particles—electrons and positives—in the system indeterminate.

* National Research Fellow.

[1] P. A. M. Dirac, Proc. Roy. Soc. **A133**, 60 (1931).

[2] E. Schroedinger, Berl. Ber., p. 63, 1931. For a review of these earlier theories of the Dirac electron, see W. Pauli, Handb. d. Physik **24**, XXIV, 242–247 (1933).

According to Dirac's theory, the states of an electron subject to no forces are given by the solutions of the wave equation

$$ih\dot{\psi} = H_0\psi; \qquad H_0 = -\beta mc^2 - c(\mathbf{a}\cdot\mathbf{p}). \quad (1.1)$$

Any solution of (1.1) may be built up out of solutions of two sorts: those of positive kinetic energy, for which

$$ih\dot{\psi} = H_0\psi = E\psi, \quad \text{with} \quad E \geq mc^2 \quad (1.2)$$

and those of negative kinetic energy:

$$ih\dot{\psi} = H_0\psi = E\psi, \quad \text{with} \quad E \leq -mc^2. \quad (1.3)$$

Any state which is a superposition of states of type (1.2) may be called positive, any of type (1.3) alone negative. Now with the help of these solutions, we can assign a matrix to represent any function of the elementary dynamical variables of the electron: p, x, α. When this matrix reduces to two submatrices for positive and negative states respectively and has no elements in which one index refers to positive, the other to negative states, the corresponding variable is said to give us an even operator. The momenta of an electron are even; the coordinates are not.

Variables which correspond to operators which are not even may still, of course, in principle be determined; important examples of such variables are the total energy of a system in an external field, and the charge and current density of a system; but this cannot be done without introducing an indeterminacy in the number of electrons and positives present. This circumstance which corresponds to a complementarity between the description of a system in terms of a definite number of permanent particles on the one hand, and a spatiotemporal specification of these particles on the other, is decisive in limiting the applicability of the nonrelativistic transformation theory of the quantum mechanics. For it is not, in general, possible to specify at the same time the number of particles in a system and the values of arbitrary dynamical variables of these particles. In particular, this means that the unambiguous determination of the presence of a positive in a system demands that under the circumstances of the experiment one should be able to abstract entirely from the creation and destruction of pairs. This must particularly be borne in mind in putting such questions as "Are there any positives present near a nucleus?"; for such a question can be given an unambiguous meaning only by reference to such an experimental arrangement for answering it, that one may abstract from the nuclear field of the atom which produces and destroys pairs.

From the formal point of view this situation implies serious limitations on the use of wave functions defined in a configuration space: wave functions whose arguments are the characteristic values of the dynamical variables of the particles of the system. For, quite apart from the limitation of the symmetry character of the wave functions imposed by the exclusion principle, we meet here for the first time the requirement that such wave functions be defined as functions only of even variables, since no physical interpretation whatever can be given to the probability that a particle shall have a determinate value for a variable which is not wholly even. The canonical transformations of the configuration space, which in nonrelativistic theory are unrestricted, are here limited to even transformations. In general, too, a wave function defined in such an even configuration space will not be adequate to describe the state of the system, since the number of particles in the system, and thus the dimensions of the configuration space, will not in general be determinate: here, as in the description of the electromagnetic field, we shall need, in general, a series of wave functions, defined in spaces of different numbers of dimensions and corresponding to the probability of finding different numbers of particles in the system. This circumstance means that, again as in the case of the electromagnetic field, the formal aspects of the theory are best studied by abandoning the wave functions defined in configuration space and by using the method of quantized waves in actual space; for specific calculations it is more convenient to revert to the configuration space functions; and it is, of course, possible to establish a correspondence between the functions introduced by the two methods.

Before we develop this formalism, we may illustrate these considerations in the important case of the position determinations of the electron. It is important to note that what we say here has to do only with the critical length $\hbar/mc$, and throws no light at all on the breakdown of the theory for lengths of the order e^2/mc^2. For what we have here to consider, the charge on the electron could have an arbitrarily small value;

and these considerations, like all those based on the present relativistic quantum mechanics, must rather be regarded as attaining asymptotic validity when the ratio of the two critical lengths, which is the fine structure constant, α, goes to zero.

It has often been observed[3] that by the gamma-ray microscope one could not, in general, determine the position of an electron with arbitrary precision. The limits on this precision are determined by the, in general, finite wave-length of light scattered through a large angle by the electron. For an electron at rest this wave-length is of the order of the Compton wave-length λ_0; for an electron whose kinetic energy before the scattering is E, it is possible to choose an arrangement such that the wave-length of the light scattered through a right angle is $\sim hc/E$. The finiteness of these wave-lengths limits, of course, the precision with which, from a given observation in the microscope, one can infer the position of the scattering electron. This limitation inheres in all attempts to localize the electron by scattering experiments, whatever be the nature of the scattered radiation.

The Dirac theory of the electron, on the other hand, starts with the postulation of a probability density $W(x)$ that the electron be found near the point x, and thus guarantees the observability of the position of the electron. But it does this only at the expense of admitting the existence of states of negative kinetic energy. For the conclusion that hard light scattered by an electron is necessarily softened by the scattering depends essentially upon the fact that the kinetic energy of the electron before the scattering is taken positive. Because of the nonexistence in fact of electrons of negative kinetic energy, the postulation of the complete localizability of the electron and the existence of the probability density $W(x)$ appears unjustifiable.

With the charge density the situation is radically different. On the Dirac theory, it is true, this charge density is merely proportional to $W(x)$:

$$\rho(x) = eW(x). \tag{1.4}$$

But for the determination of ρ other experimental procedures are available. For the quantum theory of the electromagnetic field and the careful consideration recently given by Bohr to the possibilities of observation which it implies show that, at least so far as we may abstract from the atomic nature of the measuring instruments, the electric field may be mapped out with any precision we want; its divergence gives the charge density ρ, which may by this method be in principle determined. In any theory in which the atomic nature of measuring instruments is neglected, this observability of charge density must persist. Since we have seen what grave difficulties inhere in relativistic theory in the definition of the particle density, we must be prepared to abandon the simple definition of ρ given by (1.4). As a matter of fact we shall see that in the present theory the vanishing of the mean charge density in a region of space does not necessarily mean that there are no particles present in the region; it may mean only that there are equal numbers of electrons and positives present and is no guarantee that the region is empty. For in the present theory, the charge density is not an even operator and can therefore be determined only at the expense of leaving indeterminate the number of particles present in the system.

As far as the localizability of the electron itself is concerned, the conclusions reached by the consideration of the gamma-ray microscope are fully confirmed by theory. For this it is essential to observe that the coordinate of an electron is not an even operator; this means that in the Dirac theory it is not possible at once to specify the coordinate of a particle and to be sure that its kinetic energy is positive. Now in the determination of the position of an electron, we cannot admit an indeterminacy in the number of particles present since we must be sure that the light was scattered into the microscope by the electron which we wish to observe. (We are considering for simplicity the case where only a single electron is initially present.) We cannot, therefore, devise an experiment for measuring precisely the coordinate of the electron. What we have to consider is how precisely we can infer the position of an electron from an experiment in which an even operator is determined. Thus we may determine the "even part" of the coordinate which is

[3] See for instance L. Landau and R. Peierls, Zeits. f. Physik **69**, 56 (1931).

given as an operator by

$$g(x)=x-(hc/2i)[\alpha_x H_0^{-1}+cp_z H_0^{-2}]. \quad (1.5)$$

Inferences as to the value of the coordinate depend upon the properties of the transformation functions $g\rightarrow x$. More precisely, these four-component functions are the transformation functions from g, σ the component of spin in a fixed direction and the sign of the kinetic energy to the coordinates.[4] They are given by

$$(x, i|g, \sigma, +)=2^{-\frac{1}{2}}\int_{-\infty}^{\infty} dp \exp[i(x-g)p/h]$$
$$\times[(m^2c^2+p^2)^{\frac{1}{2}}\{mc+(m^2c^2+p^2)^{\frac{1}{2}}\}]^{-\frac{1}{2}}a_i^{\sigma}, \quad (1.6)$$

where

$$a_1^1=-p, \qquad a_2^2=p,$$
$$a_3^1=mc+(m^2c^2+p^2)^{\frac{1}{2}}, \qquad a_4^2=mc+(m^2c^2+p^2)^{\frac{1}{2}},$$
$$a_2^1=a_4^1=0, \qquad a_1^2=a_3^2=0,$$

where these functions are taken for positive kinetic energy, and $\sigma=1$ corresponds to spin parallel, $\sigma=2$ to spin antiparallel, to the x axis; a set of functions for negative kinetic energy, which are relevant only in discussing the localization of the positive, may be obtained from (1.6) by replacing $(m^2c^2+p^2)^{\frac{1}{2}}$ throughout by $-(m^2c^2+p^2)^{\frac{1}{2}}$.

These functions (1.6) form a complete set for the expansion of any positive function; and the mean dispersion, defined in the usual way as $\overline{x^2}-\bar{x}^2$ vanishes for all these packets.[5] This corresponds to the circumstance that it is possible to prepare an electron in such a way that its position may be known without uncertainty. But from the results of the experiment, namely the determination of g, no unambiguous and precise inference about the position of the electron before the measurement may be drawn. For the probability of finding a value g is determined by

$$P(g)\Delta g=\Delta g \times\sum_{\sigma=1}^{2}\left|\int_{-\infty}^{\infty} dx \sum_{i=1}^{4}(g, \sigma, +|x, i)\psi_i(x)\right|^2 \quad (1.7)$$

and this is not in general given by

$$W(g)\Delta g=\Delta g\cdot\sum_{i=1}^{4}|\psi_i(g)|^2 \quad (1.8)$$

but depends upon the value of ψ in a region of the order λ_0 about g. When nothing is known about the state of the electron, ψ, before the measurement, the value of the coordinate x is fixed by the determination of g only within the limit of the Compton wave-length. When it is known that the kinetic energy of the electron before the observation is certainly as great as E, then the probability of observing a given value for g depends on the magnitude of the wave function ψ only throughout the region $g\pm hc/E$; and the position of the electron before the experiment can be inferred from the results of the experiment with a correspondingly greater precision. One may say therefore that, whereas it is possible by experiment to localize an electron, in the sense that the position of the electron after the experiment may be determined as precisely as one wishes, it is nevertheless not possible to determine the position of an electron in an arbitrary state with a precision greater than λ_0. It will be seen that this corresponds exactly to the possibilities offered by the gamma-ray microscope where the initial determination of position is necessarily unprecise by about hc/E, but where a second determination of position, after the electron is known to have scattered a hard gamma-ray, may in principle be made precise.

We have in this discussion assumed, in accordance with the physical considerations adduced and in conformity with the theory which we are here to develop, that the states of an electron in a system where it is known that no positives or pairs are produced correspond to purely positive functions; in the same way we shall see that the states of the isolated positive correspond to purely negative states; and the limits here derived for the localizability of an electron hold unchanged for a positive. In situations in which, as in the determination of the charge density, we cannot be sure of the number of charges present in our system, no unambiguous position measurements of the charges are possible.

Because of these considerations it is not in general possible, in experimental situations where electron pairs are produced, to localize the region

[4] We here give the formulae for the x-coordinate, and take $p_y=p_z=0$: analogous formulae can be given which correspond to determining $g(x)$, $g(y)$ and $g(z)$ at once.

[5] This is apparently in disagreement with the argument of Landau and Peierls, reference 3.

in which they are produced with a precision much greater than the Compton wave-length. In this sense the production of pairs by the absorption of gamma-rays and their internal conversion must be regarded as an extranuclear phenomenon; as far as present theory is concerned, the fields within the nuclei have singularly little effect upon the probability of these processes.

We must turn now to the development of the formal theory. In the next section we shall introduce the necessary methods, set up the wave equations which the wave functions of the theory must satisfy, and give the necessary proofs of invariance. In Section 3 we shall show how these equations may be solved and how their solutions are connected with those of the original Dirac theory, and justify the use of this theory in most of the cases to which it has been applied. In Section 4 we must discuss certain new effects introduced in the present theory and define those limitations on the applicability of the theory which are introduced by the here postulated and not understood corpuscular nature of the elementary charges.

2.

We must now find what wave functions describe the states and what operators correspond to the dynamical variables, in particular the energy and charge and current density, of a system of electrons and positives. In the non-relativistic quantum mechanics, a system of N indistinguishable particles which satisfy the exclusion principle may be described by an anti-symmetric wave function in the $3N$-dimensional configuration space of the particles:

$$\psi(x_1 y_1 z_1 \cdots x_N y_N z_N). \tag{2.1}$$

We can find the wave function corresponding to this state as a function of any other complete set of commuting variables $\xi_1, \xi_2 \cdots \xi_{3N}$ with the help of the transformation functions

$$(\xi_1 \cdots \xi_{3N} | x_1 \cdots z_N):$$

$$\psi(\xi_1 \cdots \xi_{3N}) = \int \cdots \int dx_1 \cdots dz_N (\xi_1 \cdots \xi_{3N} | x_1 \cdots z_N) \psi(x_1 \cdots z_N). \tag{2.2}$$

In particular we can specify the configuration of the jth electron by the three commuting variables ξ_j, η_j, ζ_j instead of x_j, y_j, z_j, and use the wave functions

$$\psi(\xi_1 \cdots \zeta_N) = \int \cdots \int dx_1 \cdots dz_N \prod_{j=1}^{N} (\xi_j \eta_j \zeta_j | x_j y_j z_j) \psi(x_1 \cdots z_N) = \psi(r_1 \cdots r_N) \tag{2.3}$$

for the system. It will in no way impair the generality of the argument to suppose the sets of characteristic values of ξ, η, ζ enumerable; we can then order these sets of characteristic values in an arbitrary but fixed order; and for any set we can use the symbol r. We can then fix the sign of the antisymmetric function (2.3) by letting its arguments be ordered. If we introduce[6] new variables N_r to describe the system, one for each characteristic value of r, and let $N_r = 1$ when there is a particle present in the state r, $N_r = 0$ when there is no particle present in this state, then we can describe the state of the system by a wave function of the N_r's, defined by

$$\psi(N_r) = 0 \quad \text{when} \quad \sum_r N_r \neq N$$

$$\psi(0 \cdots 1_{r_1} \cdots 0 \cdots 1_{r_i} \cdots 1_{r_N} \cdots) = (N!)^{\frac{1}{2}} \psi(r_1 \cdots r_N). \tag{2.4}$$

Any dynamical variable of the system which is symmetric in the particles corresponds to an operator on (2.4). To a variable which, as an operator on (2.3), is given by

$$\sum_{j=1}^{N} \Omega_{r_j' r_j} \tag{2.5}$$

there corresponds the operator

$$\sum_{r, r'} \Omega_{r' r} a_{r'}^{+} a_r. \tag{2.6}$$

The a_r, a_r^{+} here introduced satisfy

[6] P. Jordan and E. Wigner, Zeits. f. Physik **47**, 631 (1928).

$$a_r a_s + a_s a_r = 0; \quad a_r^+ a_s^+ + a_s^+ a_r^+ = 0;$$
$$a_r^+ a_s + a_s a_r^+ = \delta_{rs}. \quad (2.7)$$

They may be expressed in terms of the N_r's and operators Γ_r, v_r:

$$\Gamma_r F(N_1 \cdots N_{r-1}, N_r, N_{r+1}, \cdots) = F(N_1 \cdots N_{r-1}, 1-N_r, N_{r+1} \cdots)\Gamma_r;$$
$$v_r F(N) = (-)^{\sum_{s=1}^{r-1} N_s} F(N) \quad (2.8)$$

by

$$a_r = v_r \Gamma_r N_r; \quad a_r^+ = N_r \Gamma_r v_r. \quad (2.9)$$

Thus

$$N_r = a_r^+ a_r; \quad 1 - N_r = a_r a_r^+;$$
$$N_r^2 = N_r; \quad \Gamma_r^2 = 1; \quad v_r^2 = 1. \quad (2.10)$$

Similarly, to an operator on (2.3) of the form

$$\sum_{i,j} \Omega_{r_i' r_j';\, r_i r_j} \quad (2.11)$$

there corresponds the operator

$$\sum_{r', s', r, s} \Omega_{r's';\, rs} a_{r'}^+ a_r a_{s'}^+ a_s \quad (2.12)$$

and to

$$\sum_{i \neq j} \Omega_{r_i' r_j';\, r_i r_j} \quad (2.13)$$

the operator

$$\sum_{r', s', r, s} \Omega_{r's';\, rs} a_{r'}^+ a_{s'}^+ a_r a_s. \quad (2.14)$$

It is in terms of this formalism that the relativistic theory of electrons and positives may most easily be developed. The formalism as it stands is immediately applicable to the Dirac theory of the electron in its original form and describes a system of N electrons whose wave functions are given by the Dirac wave equation, and in which the electrons may have positive or negative kinetic energies. (To describe the state of an electron, r must be supposed to include a variable which determines the orientation of the spin of the electron.) But a simple formal change, which corresponds exactly to regarding the emptiness and not the fullness of a state of negative kinetic energy as equivalent to the presence of a particle, gives us at once the theory of electrons and positives. To make this change it is necessary, in accordance with the physical considerations of Section 1, to restrict the r's in a critical way. From now on we shall consider only states r, such that a given state is either wholly positive or wholly negative; the states r which are positive shall be indexed by Latin letters; the negative by Greek; and a state which may be either positive or negative will be indexed by a Latin letter in parentheses: (r). By this restriction the unlimited applicability of the quantum-mechanical transformation theory is destroyed.

For what follows it is helpful to generalize our definition of positive and negative states. A positive state is one which is a superposition of states of a free particle of positive kinetic energy. And the states of a free particle are determined by a wave equation which is in general

$$(c\pi_0 - T)\psi = 0; \quad T = -c(\boldsymbol{\alpha} \cdot \boldsymbol{\pi}) - \beta mc^2. \quad (2.15)$$

Here

$$\pi_0 = (ih/c)(\partial/\partial t) - (e/c)V;$$
$$\boldsymbol{\pi} = -ih \operatorname{grad} - (e/c)\mathbf{A} \quad (2.16)$$

and, for no field

$$V = -(1/c)(\partial/\partial t)\lambda(x, y, z, t);$$
$$\mathbf{A} = \operatorname{grad} \lambda(x, y, z, t).$$

The solutions of (2.15) are given in terms of the solution ψ_0 of (2.15) with $\lambda = 0$ by the transformation

$$\psi = \psi_0 \exp(ie\lambda/\hbar c).$$

We have of course to consider solutions of (2.15) which correspond to definite values of the kinetic energy, which is represented by the operator $c\pi_0 = T$. It will be observed that under the transformation

$$V' = V - \frac{1}{c}\frac{\partial \lambda}{\partial t}; \quad \mathbf{A}' = \mathbf{A} + \operatorname{grad} \lambda;$$
$$\psi' = \psi \exp(ie\lambda/hc) \quad (2.17)$$

the kinetic energy is invariant; under it positive states go into positive, negative into negative states.

Let us consider an operator of the form (2.5) for which $\Omega_{(r)(s)}$ is a diagonal matrix: to it will correspond the operator

$$\sum_r \Omega_{rr} N_r + \sum_\rho \Omega_{\rho\rho} N_\rho \quad (2.18)$$

or, if we write $M_\rho = 1 - N_\rho$,

$$\sum_r \Omega_{rr} N_r - \sum_\rho \Omega_{\rho\rho} M_\rho + \sum_\rho \Omega_{\rho\rho}. \quad (2.19)$$

For a system containing N electrons—i.e., for which N of the N_r for the occupied states are 1, and the others vanish—and containing M positives, in states corresponding to $\rho_1 \cdots \rho_M$, we should expect for the operator (2.6) just

$$\sum_r \Omega_{rr} N_r + \sum_\rho (-\Omega_{\rho\rho}) M_\rho. \qquad (2.20)$$

We are thus led to consider, in connection with an arbitrary operator Ω, the operator

$$\hat{\omega} = \sum_{(r)(s)} \Omega_{(r)(s)} \omega_{(r)(s)} \qquad (2.21)$$

with

$$\omega_{r(s)} = a_r^+ a_{(s)}; \quad \omega_{(r)s} = a_{(r)}^+ a_s;$$
$$\omega_{\rho\sigma} = a_\rho^+ a_\sigma - \delta_{\rho\sigma} = -a_\sigma a_\rho^+. \qquad (2.22)$$

This assignment suffices for a consistent description of a system of noninteracting electrons and positives. To the question of the treatment of the interaction of these particles, which cannot be wholly clarified, and to which no complete and unambiguous answer can be given, we shall return only later.

The energy of a system of noninteracting particles, which may however be subject to arbitrary external forces, is of the form (2.5), and corresponds then to the operator

$$\underset{\sim}{H} = \sum_{(r)(s)} H_{(r)(s)} \omega_{(r)(s)}. \qquad (2.23)$$

In particular the kinetic energy $\mathfrak{T}$ of the system is

$$\sum_r N_r T_{rr} - \sum_\rho M_\rho T_{\rho\rho} \qquad (2.24)$$

and is necessarily positive, the sum of the positive kinetic energies of the electrons and the positive kinetic energies $-T_{\rho\rho}$ of the positives. The difference between the total number of positives and electrons in the system, which gives the total charge,

$$-e\nu = e(M-N); \quad M = \sum_\rho M_\rho; \quad N = \sum_r N_r$$

commutes with $\underset{\sim}{H}$, and is constant, as it should be: to any system there corresponds a fixed value of ν. But both M and N change with time in general:

$$-ih\dot{N} \equiv -ih\dot{M} \equiv \sum_{r\rho} (H_{\rho r} a_\rho^+ a_r - H_{r\rho} a_r^+ a_\rho). \qquad (2.25)$$

Unless the energy corresponds to an even operator the number of particles in the system $N+M$ will not be constant. The rate of production of a pair r, ρ is measured by $|H_{r\rho}|^2$ and that of its annihilation by $|H_{\rho r}|^2$.

The modification by which (2.21) was obtained from (2.6), namely the subtraction of

$$\sum_-(\Omega) = \sum_\rho \Omega_{\rho\rho} \qquad (2.26)$$

has destroyed, as we have seen, and as the physics requires, the invariance of the theory under arbitrary contact transformations. If we restrict ourselves however to the even transformations, $((r)|(s))$, for which $((r)|(s))$ is an even matrix, then $\sum_-(\Omega)$, which is the spur of the matrix $\Omega_{(r)(s)}$ in an invariant subspace, will remain invariant. To any wave function of the N_r and M_ρ there will correspond a wave function of N_s and M_σ for any s, σ which are obtained from r, ρ by an even transformation; and under such transformation the theory is invariant.

The Lorentz invariance of the theory follows from this. For under Lorentz transformation positive kinetic energies go over into positive, negative into negative; the Lorentz transformation is even. Thus

$$\sum_-(\Omega) \quad \text{and} \quad \hat{\omega} = \sum_{(r)(s)} \Omega_{(r)(s)} a_{(r)}^+ a_{(s)} - \sum_-(\Omega)$$

will transform under a Lorentz transformation just as Ω does; if Ω is a scalar, then

$$\sum_-(\Omega) \quad \text{and} \quad \hat{\omega}$$

will be scalars; if the Ω^μ form a four vector, so will the

$$\sum_-(\Omega^\mu) \text{ and the } \hat{\omega}^\mu.$$

In the present formulation the subtraction of the spur (2.26) destroys the gauge invariance of the theory, in that it is not possible to associate in an unambiguous way with a given choice of gauge to represent a given field a unique gauge in (2.16). This circumstance means that the division of states into positive and negative here formulated is still provisional, and will have to be modified in the presence of an electromagnetic field. As the arguments of the next section will show, the non-gauge-invariance of the theory does not affect the predictions which the theory makes about the reactions of the system to a given external field—the energy differences of the states of the system in the field, the transition probabilities, and the probabilities of production of observable pairs. On the other hand, the predictions of the theory about the reaction of the system on the electromagnetic field are gauge-dependent.

Again with neglect of the interaction of the particles, we may write the wave equation for the functions $\psi(N_r; M_\rho)$. If there is an electromagnetic field described by the potentials V, A then the Hamiltonian is given by

$$H=eV+ihc(\mathfrak{a}, \text{grad})+e(\mathfrak{a}\cdot A)-\beta mc^2. \quad (2.27)$$

To this there corresponds the matrix $H_{(r)(s)}$ and to this the operator

$$\underline{H}=\sum_{(r)(s)} H_{(r)(s)}\omega_{(r)(s)}. \quad (2.28)$$

The wave equation is then

$$ih(\partial/\partial t)\psi(N_r; M_\rho)=\underline{H}\psi(N_r; M_\rho). \quad (2.29)$$

We can readily write the wave equation in the configuration space of the particles which corresponds to this. We define the wave functions in configuration space, in analogy to (2.4), by[7]

$$\psi(N_r; M_\rho)=0, \qquad N-M\neq\nu$$
$$\psi(0\cdots 1_{r_1}0\cdots 1_{r_2}\cdots 1_{r_N}\cdots; 0\cdots 1_{\rho_1}\cdots 0\cdots 1_{\rho_M}\cdots)=(N!M!)^{\frac{1}{2}}(-)^{\sum_j[\rho_j]}\psi_{NM}(r_1\cdots r_N; \rho_1\cdots\rho_M),$$
$$N-M=\nu \quad (2.30)$$

with the arguments of the ψ_{NM} ordered. We may now define the operator $\hat{\omega}$ corresponding to (2.5) and (2.21) as an operator in configuration space. This operator transforms any function ψ into $\hat{\omega}\psi$, according to

$$(\hat{\omega}\psi)_{N,M}(r;\rho)=\sum_{r'}\sum_{i}^{N}\Omega_{r_i r'}\psi_{N,M}(\cdots r_{i-1}r'r_{i+1}\cdots;\rho)-\sum_{\rho'}\sum_{j}^{M}\Omega_{\rho'\rho_j}\psi_{N,M}(r;\cdots\rho_{j-1}\rho'\rho_{j+1}\cdots)$$
$$-(NM)^{-\frac{1}{2}}\sum_{i}^{N}\sum_{j}^{M}(-)^{N-i+j-1}\Omega_{r_i\rho_j}\psi_{N-1,M-1}(\cdots r_{i-1}r_{i+1}\cdots;\cdots\rho_{j-1}\rho_{j+1}\cdots)$$
$$-(N+1)^{\frac{1}{2}}(M+1)^{\frac{1}{2}}\sum_{r'}\sum_{\rho'}\Omega_{\rho' r'}\psi_{N+1,M+1}(r_1\cdots r_N r';\rho'\rho_1\cdots\rho_M). \quad (2.31)$$

Then the wave equation is

$$ih(\partial/\partial t)\psi_{N,M}(r;\tau)=(\underline{H}\psi)_{N,M}(r;\rho) \quad (2.32)$$

or, in detail, for $N-M=\nu=0$:

$$ih(\partial/\partial t)\psi_{00}=-\sum_{r'}\sum_{\rho'}H_{\rho' r'}\psi_{11}(r';\rho'), \quad (2.33)$$

$$ih(\partial/\partial t)\psi_{11}(r_1;\rho_1)=\sum_{r'}H_{r_1 r'}\psi_{11}(r';\rho_1)-\sum_{\rho'}H_{\rho'\rho_1}\psi_{11}(r_1;\rho')-H_{r_1\rho_1}\psi_{00}-2\sum_{r'}\sum_{\rho'}H_{\rho' r'}\psi_{22}(r_1r';\rho'\rho_1);$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

for $\nu=1$:

$$ih(\partial/\partial t)\psi_{10}(r_1)=\sum_{r'}H_{r_1 r'}\psi_{10}(r')-2^{\frac{1}{2}}\sum_{r'}\sum_{\rho'}H_{\rho' r'}\psi_{21}(r_1r';\rho'),$$
$$ih(\partial/\partial t)\psi_{21}(r_1r_2;\rho_1)=\sum_{r'}H_{r_1 r'}\psi_{21}(r'r_2;\rho_1)+\sum_{r'}H_{r_2 r'}\psi_{21}(r_1r';\rho_1)$$
$$-\sum_{\rho'}H_{\rho'\rho_1}\psi_{21}(r_1r_2;\rho')-2^{-\frac{1}{2}}H_{r_2\rho_1}\psi_{10}(r_1)+2^{-\frac{1}{2}}H_{r_1\rho_1}\psi_{10}(r_2)-6^{\frac{1}{2}}\sum_{r'}\sum_{\rho'}H_{\rho' r'}\psi_{32}(r_1r_2r';\rho'\rho_1); \quad (2.34)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

and for $\nu=-1$:

$$ih(\partial/\partial t)\psi_{01}(\rho_1)=-\sum_{\rho'}H_{\rho'\rho_1}\psi_{01}(\rho')-2^{\frac{1}{2}}\sum_{r'}\sum_{\rho'}H_{\rho' r'}\psi_{12}(r';\rho'\rho_1),$$
$$ih(\partial/\partial t)\psi_{12}(r_1;\rho_1\rho_2)=\sum_{r'}H_{r_1 r'}\psi_{12}(r';\rho_1\rho_2)-\sum_{\rho'}H_{\rho'\rho_1}\psi_{12}(r_1;\rho'\rho_2)$$
$$-\sum_{\rho'}H_{\rho'\rho_2}\psi_{12}(r_1;\rho_1\rho')-2^{-\frac{1}{2}}H_{r_1\rho_1}\psi_{01}(\rho_2)+2^{-\frac{1}{2}}H_{r_1\rho_2}\psi_{01}(\rho_1)-6^{\frac{1}{2}}\sum_{r'}\sum_{\rho'}H_{\rho' r'}\psi_{32}(r_1r';\rho'\rho_1\rho_2). \quad (2.35)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

[7] Here $[\rho_j]$ is the ordinal number assigned to the state ρ_j in ordering the negative states. The factor $(-)^{\sum_j[\rho_j]}$ is inserted to save writing.

The rule for the transition from (2.6) to (2.21) cannot be unambiguously extended to operators of the form (2.14); and for the consideration of the electromagnetic interaction of the particles of the system we must revert to electrodynamical theory. According to (2.21), to the charge density

$$\sum_i \rho_{(r)_i'(r)_i}; \quad \rho_{(r)'(r)} = e\bar{\psi}_{(r)'}\psi_{(r)} \tag{2.36}$$

and current density

$$\sum_i \mathbf{j}_{(r)_i'(r)_i}; \quad j_{(r)'(r)} = e\bar{\psi}_{(r)'}\boldsymbol{\alpha}\psi_{(r)} \tag{2.37}$$

must be assigned the operators

$$\sum_{(r)(s)} \rho_{(r)(s)}\omega_{(r)(s)} \quad \text{and} \quad \sum_{(r)(s)} \mathbf{j}_{(r)(s)}\omega_{(r)(s)}. \tag{2.38}$$

The form a four vector $\breve{S}^\mu$.

The electrostatic interaction of the system with itself, i.e., the Coulomb forces exerted by the particles on themselves and on each other, may then, with neglect of retardation, be obtained from the matrix

$$C_{(r)(s);\,(l)(m)} = \int\int d\tau d\tau' \frac{\rho_{(r)(s)}(x)\rho_{(l)(m)}(x')}{|r-r'|} \tag{2.39}$$

and give an operator

$$\hat{C} = \sum_{(r),\,(s),\,(l),\,(m)} C_{(r)(s):\,(l)(m)}\omega_{(r)(s)}\omega_{(l)(m)}. \tag{2.40}$$

This includes of course in its present form the proper electrostatic energy of the particles.

When the magnetic forces and the retardation of the forces between the charges must be taken strictly into account, there can be, of course, no operator in the configuration space of the particles to represent the corresponding energy; rather one must use the expressions (2.38) for the charge and current of the system, and from these compute the interaction energy between the charges and the electromagnetic field

$$H_I = \int d\tau[\rho V + (\mathbf{j}\cdot\mathbf{A})]. \tag{2.41}$$

This expression may then be treated as a perturbation which induces an interchange of energy between the particles and the field; and in its application one meets only the same difficulties as in the Dirac dispersion theory: convergence difficulties which rest ultimately on an illegitimate application of the methods of the quantum mechanics to the electromagnetic field.

In addition to the approximation (2.39) by which, with the total neglect of retardation, one has the possibility of taking into account to all orders the Coulomb interactions of the charges, one has had, as is known, a second method of approximation by which one can take the fully retarded interaction into account to the first order only. For the case that, in the absence of interactions, the Hamiltonian of the system is even and has the characteristic values E_r, E_ρ, the interaction may to the first order be taken into account by the inclusion of a term in the energy which corresponds to the operator

$$\bar{A} = \tfrac{1}{2}\sum_{(r)(s)(l)(m)} A_{(r)(s):\,(l)(m)}\omega_{(r)(s)}\omega_{(l)(m)};$$

$$A_{(r)(s):\,(l)(m)} = \tfrac{1}{2}\int\int d\tau d\tau' \sum_{\mu=1}^{4} \frac{s^\mu_{(r)(s)}(x)s^\mu_{(l)(m)}(x')}{|r-r'|}[\exp(i\nu_{(r)(s)}r/c)+\exp(i\nu_{(l)(m)}r/c)] \tag{2.42}$$

with $\nu_{(r)(s)} = (E_{(r)} - E_{(s)})/\hbar$. But the application of this method is seriously limited on the present theory. For an unambiguous extension of this to the case where the odd parts of the Hamiltonian without interaction are not negligible does not appear to be possible, because the method presupposes that in the absence of interactions the particles remain in stationary states, and, on the present theory, this is not, in general, true; in general, namely, there will be no even states r which give determinate values to the energy of the system.

The interactions between the particles, in particular, the Coulomb interactions given by (2.40), can certainly not be entirely neglected, since one must surely expect that positives and electrons will attract and repel each other. On the other hand, there are ambiguities in the application of (2.40), which arise from the apparent impossibility of drawing a rational distinction in all cases between proper energy and interaction energy, and which we must now discuss. Two consequences in particular of these interactions must be noted. In the first place, they give rise to the possibility of the simultaneous production or destruction of more than one pair, a possibility which does not, of course, exist when they are neglected. In the second place they introduce deviations in the energy levels of an electron in a static field computed from Dirac's equation. These effects are small, and in view of the ambiguities involved in applying (2.40), it appears that the application of the formalism to these problems transcends the limits within which

the methods are valid. For these ambiguities can apparently be dissipated only by a clearer understanding of the stability of the elementary particles.

The terms in (2.40) with $C_{rs;\,lm}$, $C_{\rho\sigma;\,\lambda\mu}$ and $C_{rs;\,\rho\sigma}$ correspond to the even parts of the interactions of electrons with electrons, positives with positives, and electrons with positives. In analogy to the replacement of (2.12) by (2.14), we may eliminate the proper energies by

$$\omega_{rs}\omega_{lm}\rightarrow a_r{}^+a_l{}^+a_ma_s, \quad \omega_{\rho\sigma}\omega_{\lambda\mu}\rightarrow a_\sigma a_\mu a_\lambda{}^+a_\rho{}^+,$$

leaving

$$\omega_{rs}\omega_{\rho\sigma}=-a_r{}^+a_sa_\sigma a_\rho{}^+ \text{ and } \omega_{\rho\sigma}\omega_{rs}=-a_\sigma a_\rho{}^+a_r{}^+a_s.$$

Analogous rules can be given for all the other terms of (2.40)

$$\omega_{rs}\omega_{l\mu}\rightarrow a_r{}^+a_l{}^+a_\mu a_s, \quad \omega_{\mu r}\omega_{sl}\rightarrow a_\mu{}^+a_s{}^+a_la_r,$$
$$\omega_{\rho\sigma}\omega_{m\lambda}\rightarrow a_\sigma a_\lambda a_m{}^+a_\rho{}^+, \quad \omega_{\rho m}\omega_{\sigma\lambda}\rightarrow a_ma_\lambda a_\sigma{}^+a_\rho{}^+.$$
$$\omega_{\rho r}\omega_{s\sigma}\rightarrow a_\sigma a_\rho{}^+a_s{}^+a_r.$$

The remaining terms should not be changed. These changes have been made in such a way that no infinite contributions to the electrostatic energy appear. Because there is no equation in configuration space with which to compare our formulae, we cannot be sure that as far as the odd parts of (2.40) are concerned our results are right: they lead to no grossly incorrect consequences. In fact the distinction between proper energy and interaction energy, which has in the present state of field theory to be made, cannot be made without ambiguity in a system in which the number of particles is necessarily indeterminate.

3.

The wave functions which we have introduced, $\psi_{N,\,M}(r;\rho)$, give us directly the probability $P(r_1\cdots r_N;\rho_1\cdots\rho_M)$ of finding in the system N electrons and M positives in the states $r_1\cdots\rho_M$. Just as in the case of the general transformation functions of nonrelativistic theory, it is here not always possible to say to what extent these probabilities can be determined by actual experiment. For although the theory enables us to devise conceptual experiments for the determination of P which in no way contradict the theoretical possibilities, it is not in general possible to find actual physical means for carrying them out. This may be illustrated by considering the function ψ for a static, non-even field, and taking the (r) to be the components of momentum of the particles. Here, by suddenly destroying the field, destroying it, that is, in a time very short compared to h/mc^2, and determining in the usual way the momenta of the free particles, we can in principle determine P. Again, by scattering hard radiation, and observing the momenta of the particles of the pair and of the light after scattering, we could in principle determine the original momentum distribution with the same precision as in nonrelativistic atomic theory. But here an important point must be noted: the gamma-ray—or electron—which we use for the experiment, will itself tend to produce or destroy pairs; for the perturbation it induces is represented by an operator which is not even; and to determine the initial distribution in momentum of the pairs, we should have actually to use an "even" gamma-ray. It is clear therefore that the actual experiments in which positives are observed do not provide us with unambiguous determinations of P for the field around nuclei. It is clear too that the theory in its present form does not impose restrictions on the available experimental means, which make P unobservable; from the point of view of this theory, there could be such a thing as an "even" gamma-ray. Nor is it possible to say whether the limitations which are in fact imposed ought to be recognized by a consistent theory: whether, that is, they should be regarded as limitations in principle or by accident. It seems clear that such a limitation in the experimental possibilities could only be based on a theory in which the electromagnetic nature of the electrons and positives were understood; and it is in the sense of the quantum mechanical theory, and the approximation $\alpha\rightarrow 0$ which defines its applicability, that these restrictions should not find a counterpart in the theory, or the possibilities of observation which it offers. This situation, however, makes it particularly urgent to show how the wave functions $\psi_{N,\,M}(r;\rho)$, which are only in principle and hardly in fact observable, can be used for the calculation of things that the experiments do give us.

When the interaction of the particles can be neglected, the wave equation (2.32) determines the characteristic values of the energy of the system in static fields, and the transition probabilities in collisions and radiative processes. Now to certain of these questions, such as the energy levels of an electron in some static fields, or the probability of scattering of light by an electron, the Dirac theory of the electron gives us an answer, and one in large measure confirmed by experiment. We have now to show how, in these

cases, the new formalism gives the same answer as the Dirac theory.

Let us consider first the characteristic values of the energy of an electron in a static field. According to the Dirac theory these are determined as the characteristic values of H in the equation

$$Hu_n = E_n u_n. \tag{3.1}$$

Under certain circumstances, some of these characteristic values give correctly the energy levels of an electron in the field of force, and the corresponding wave functions may be used to describe the behavior of the electron in its stationary states. The circumstances under which this is so may be defined in the following way: if we imagine the field in which the electron is moving (or the odd part of the field) to go adiabatically to zero, the wave functions will go over into the wave functions of a free particle (or the functions for a particle in an even field of force). In the absence of degeneracies of a special kind, any solution u_n will go over into a positive or purely negative function. A function which in this sense is "genetically positive" (L) will correspond to a possible state of the electron, and the energy value E_L to a possible energy level. The functions Λ which are genetically negative do not correspond to states of the electron. Such an unambiguous application of the adiabatic theorem will, in general, only then be possible, when the stationary states corresponding to the even part of the energy are not in this sense degenerate, that positive and negative functions correspond to the same energy value. When, as in the problem of the Klein paradox, such degeneracies occur, no simple interpretation of the Dirac wave functions can be given. But in many problems, in particular in the problems of electrons in atomic fields, and in general in all stationary state problems to which the Dirac theory gives correct answers, the classification of states as genetically positive (L) or genetically negative (Λ) will be possible; and it is just in these cases that we can establish a connection between the wave functions of the present theory and those of the Dirac theory in its original form, and a corresponding equivalence of energy levels. Without in general investigating the properties of the fields for which this division of states is possible, we may remark that of the simple fields defined by a scalar potential which is a polynomial in r and $1/r$, the Coulomb field alone[8] permits this classification. As for the states Λ, which in the Dirac theory are simply rejected, we shall see that in the present theory, as we should expect, these define the states of the positive in the field, and give $-E_\Lambda$ for the corresponding energy levels.

In the present formalism the energies E_L, E_Λ, the wave functions u_L and u_Λ, do not appear directly at all. Rather (2.33) determines the states of the system when it is known that there are just as many electrons as positives present; (2.34) when it is known that there is one electron more; (2.35) when there is one extra positive, and so on. And the energies E_L must be connected with the differences in energy levels of (2.34) and (2.33), those of the positive with differences between (2.35) and (2.33). Now just in the case of fields for which, on the Dirac theory, the classification of states as L or Λ is possible, there is a unique state (O), for which the energy is a minimum, and which corresponds to the normal state of the pairs in that field. The interpretation of the wave function for this state has already been discussed; to the question of the interpretation of the energy we shall return in the next section. Let us call this energy $E^{(O)}$, and the wave functions for this state $\psi^{(O)}$. For $\nu=1$, (2.34) will give us a set of energies $E^{(L)}$ and corresponding wave functions; for $\nu=-1$, we shall get the energies $E^{(\Lambda)}$. We shall now show that the differences between the $E^{(L)}$ and $E^{(O)}$ are just the L energy values given us by the Dirac equation:

$$E^{(L)} - E^{(O)} = E_L \tag{3.2a}$$

and similarly that the differences between the $E^{(\Lambda)}$ and $E^{(O)}$ are the negative of the Λ energy values of the Dirac equation:

$$E^{(\Lambda)} - E^{(O)} = -E_\Lambda. \tag{3.2b}$$

In particular the energy differences $E^{(L)} - E^{(L')}$ are the energy differences $E_L - E_{L'}$. This result is essential, for it shows that where the Dirac theory gives correct energy levels for an electron, the present theory gives the same energies. In particular it shows that the present theory will give again Sommerfeld's formula for the relativistic doublets. In the derivation of the result it is

[8] M. S. Plesset, Phys. Rev. **41**, 278 (1932).

essential that we neglect the effect of the electromagnetic interaction of the particles of the pairs.[9]

For the proof of (3.2) one can on the one hand use perturbation methods, treating the odd parts of H as small. If we let the states r, ρ diagonalize the even part of H, we find, to the second order in $H_{r\rho}$: for $\nu=0$:

$$E^{(0)}=-\sum_{r,\rho}|H_{r\rho}|^2/(E_r-E_\rho); \tag{3.3}$$

for $\nu=1$:

$$E^{(l)}=-\sum_{r\neq l}\sum_{\rho}|H_{r\rho}|^2/(E_r-E_\rho); \tag{3.4}$$

and for $\nu=-1$

$$E^{(\lambda)}=-\sum_{r}\sum_{\rho\neq\lambda}|H_{r\rho}|^2(E_r-E_\rho). \tag{3.5}$$

The differences between (3.4) and (3.3) on the one hand, and (3.3) and (3.5) on the other, are however just the terms of this order in E_L and E_Λ for those states L and Λ which go over into l, λ when the odd part of H vanishes. In interpretation of this result it may be remarked that that contribution to the energy which in the Dirac theory may formally be ascribed to transitions to states of negative kinetic energy, here appears because of the interdiction, on introduction of the electron, of the formation of such pairs as have electrons in the state in which the electron was introduced. This perturbation method can be extended to prove (3.2) in all orders, but soon becomes unwieldy.

We may obtain a more powerful method of solving the wave equations (2.32) by the following construction. We introduce the components of the Dirac wave functions $u_{(L)}$ in the (r) space:

$$C_{(r)}{}^{(L)}=\int d\tau\bar{u}_{(r)}u_{(L)} \tag{3.6}$$

and form the wave functions

$$\begin{aligned}\psi^{((L);\ (0)}_{N+1,\ N}(r;\rho)&=(N+1)^{-\frac{1}{2}}\sum_{i=1}^{N+1}(-)^{i-1}C_{r_i}{}^{(L)}\psi^{(0)}{}_{N,\ N}(\cdots r_{i-1}r_{i+1}\cdots;\rho)\\&\quad+(N+1)^{\frac{1}{2}}\sum_{\rho'}C_{\rho'}{}^{(L)}\psi^{(0)}_{N+1,\ N+1}(r;\cdots\rho_N\rho'),\\ \psi^{(0;\ (L))}_{N,\ N+1}(r;\rho)&=(N+1)^{-\frac{1}{2}}\sum_{i=1}^{N+1}(-)^{N+1-i}\bar{C}_{\rho_i}{}^{(L)}\psi^{(0)}{}_{N,\ N}(r;\cdots\rho_{i-1}\rho_{i+1}\cdots)\\&\quad-(N+1)^{\frac{1}{2}}\sum_{r'}\bar{C}_{r'}{}^{(L)}\psi^{(0)}_{N+1,\ N+1}(r'r_1\cdots;\rho).\end{aligned} \tag{3.7}$$

It may be shown then that $\psi^{((L);\ (0)}$ and $\psi^{(0;\ (L))}$ satisfy (2.34) and (2.35), respectively, with energy values respectively $E^{(0)}+E_{(L)}$ and $E^{(0)}-E_{(L)}$. The proof is straightforward and gives us, for cases to which the Dirac equation is directly applicable, a method of constructing the wave functions of the present theory. Here all the solutions of the Dirac equation appear: the solutions L give us the wave functions for an electron, the solutions Λ those for a positive.

We must of course demand that

$$\psi^{(\Lambda;\ 0)}_{N+1,\ N}=0;\quad \psi^{(0;\ L)}_{N,\ N+1}=0, \tag{3.8}$$

for these functions would correspond to the addition of an electron in a genetically negative state or a positive corresponding to a genetically positive state. It is easy to see by direct inspection that (3.8) must be satisfied; and we obtain a proof by constructing the $\psi^{(0)}{}_{N,\ N}$ in terms of the $C^{(L)}{}_{(r)}$. This construction of the $\psi^{(0)}{}_{N,\ N}$ is suggested by the original formulation which Dirac gave to the present theory, according to which "all negative energy states" should normally be full. Alternatively we may characterize the state (0) by saying that all the "negative energy states" of the positive are full. If then we are given a complete set of solutions u_L and u_Λ and form the quantities $C^L{}_{(r)}$, $C^\Lambda{}_{(r)}$ for all these states, we may write the solutions as infinite determinants, as follows:

$$\begin{aligned}\psi^{(0)}{}_{0,\ 0}&=|\bar{C}_r{}^L|;\\ \psi^{(0)}{}_{1,\ 1}(r_1;\rho_1)&=|\bar{C}_{r_1}{}^L\to\bar{C}_{\rho_1}{}^L|,\end{aligned} \tag{3.9}$$

[9] Cf. Section 4.

and generally

$$\psi^{(O)}{}_{N,\,N}(r_1\cdots r_N;\,\rho_1\cdots\rho_N) = (N!)^{-1}\,|\,\bar{C}_{r_i}{}^{L}\to\bar{C}^{L}{}_{\rho_{N+1-i}}\,|\,, \tag{3.9}$$

where, in these determinants, in all the rows L the $\bar{C}_{r_i}{}^{L}$ are replaced by the $\bar{C}^{L}{}_{\rho_{N+1-i}}$; here r's and ρ's should be ordered. Alternatively we may write

$$\psi^{(O)}{}_{O,\,O} = |\,C_{\rho}{}^{\Lambda}\,|\,;\quad \psi^{(O)}{}_{1,\,1}(r_1;\,\rho_1) = -\,|\,C_{\rho_1}{}^{\Lambda}\to C_{r_1}{}^{\Lambda}\,|\,, \tag{3.10}$$

$$\psi^{(O)}{}_{N,\,N}(r_1\cdots r_N;\,\rho_1\cdots\rho_N) = (-)^{N}(N!)^{-1}\,|\,C_{\rho_i}{}^{\Lambda}\to C^{\Lambda}{}_{r_{N+1-i}}\,|\,.$$

The $\psi^{(O)}$ so defined, if we avoid questions introduced by the convergence of the determinants, formally satisfies (2.32), with the energy value

$$E^{(O)} = -\sum_{r,\,\rho} H_{\rho r}\psi^{(O)}{}_{1,\,1}(r;\,\rho)/\psi^{(O)}{}_{O,\,O}. \tag{3.11}$$

From these expressions for $\psi^{(O)}$ the vanishing of $\psi^{(O;\,L)}$ and $\psi^{(\Lambda;\,O)}$ follows.

We may now generalize (3.7) to give us the wave functions which correspond to the addition of n electrons in the states $L_1\cdots L_n$ and m positives in the states $\Lambda_1\cdots\Lambda_m$. These functions vanish unless all the L's and all the Λ's are distinct:

$$\begin{aligned}
\psi_{N,\,M}^{(L_1\cdots L_n(L');\ \Lambda_1\cdots\Lambda_m)}(r_1\cdots r_N;\,\rho_1\cdots\rho_M) &= N^{-\frac{1}{2}}\sum_{i=1}^{N}(-1)^{i-1}C_{r_i}{}^{(L')}\psi_{N-1,\,M}^{(L_1\cdots L_n;\ \Lambda)}(\cdots r_{i-1}r_{i+1}\cdots;\,\rho)\\
&\quad +(-)^{n+m}(M+1)^{\frac{1}{2}}\sum_{\rho'}C_{\rho'}{}^{(L')}\psi_{N,\,M+1}^{(L_1\cdots L_n;\ \Lambda)}(r;\,\cdots\rho_M\rho')\\
&= 0 \text{ unless } N-M=n+1-m;\qquad (3.12)\\
\psi_{NM}^{(L_1\cdots L_n;\ \Lambda_1\cdots\Lambda_m(L'))}(r_1\cdots r_N;\,\rho_1\cdots\rho_M) &= M^{-\frac{1}{2}}\sum_{i=1}^{M}(-)^{M-i}\bar{C}_{\rho_i}{}^{(L')}\psi_{N,\,M-1}^{(L;\ \Lambda_1\cdots\Lambda_m)}(r;\,\cdots\rho_{i-1}\rho_{i+1}\cdots)\\
&\quad -(-)^{n+m}(N+1)^{\frac{1}{2}}\sum_{r'}\bar{C}_{r'}{}^{(L')}\psi_{N+1,\,M}^{(L;\ \Lambda_1\cdots\Lambda_m)}(r'r_1\cdots;\cdot\rho)\\
&= 0 \text{ unless } N-M=n-m-1.
\end{aligned}$$

The identities corresponding to (3.8) are now of the form

$$\psi^{(L_1\cdots L_n\Lambda';\ \Lambda_1\cdots\Lambda_m)}=0;\qquad \psi^{(L_1\cdots L_n;\ \Lambda_1\cdots\Lambda_m L')}=0 \tag{3.13}$$

and follow, as before, from (3.9). In this way we can construct a series of solutions, corresponding to different values of n, for all of which $n=m$ and $\nu=0$. The transition from a state n to $n+1$ corresponds to the creation of a pair; that from n to $n-1$ to its annihilation. Similarly the transition, with $m=0$, $n=1$, from the state L to L' gives just the transition of the electron from one "genetically positive" state to another, and that from Λ to Λ', with $m=1$, $n=0$, gives the corresponding transition for the positives.

With the help of the functions (3.12) we can establish another useful result. Let us consider an operator of the form (2.5). To this there will correspond an operator $\hat{\omega}$ on the $\psi(N_r;\,M_\rho)$, and from this, according to (2.31), we can construct the corresponding operator as a matrix in the scheme L, Λ. The only elements of this matrix which do not vanish are those for which $(L)\to(L')$, $(\Lambda)\to(\Lambda')$, $(L;\Lambda)\to(O)$, or $(O)\to(L;\Lambda)$. By using the functions (3.12) we can now prove that

$$\begin{aligned}
(L\,|\,\hat{\omega}\,|\,L') &= (\bar{\psi}^{(L)}\cdot\hat{\omega}\psi^{(L')}) = \Omega_{L\,L'} = \int d\tau\bar{u}_L\Omega u_{L'};\\
(\Lambda\,|\,\hat{\omega}\,|\,\Lambda') &= (\bar{\psi}^{(\Lambda)}\cdot\hat{\omega}\psi^{(\Lambda')}) = -\,\Omega_{\Lambda'\Lambda} = -\int d\tau\bar{u}_{\Lambda'}\Omega u_{\Lambda};\\
(L;\,\Lambda\,|\,\hat{\omega}\,|\,O) &= (\bar{\psi}^{(L;\ \Lambda)}\cdot\hat{\omega}\psi^{(O)}) = -\,\Omega_{L\Lambda} = -\int d\tau\bar{u}_L\Omega u_{\Lambda};\\
(O\,|\,\hat{\omega}\,|\,L;\,\Lambda) &= (\bar{\psi}^{(O)}\cdot\hat{\omega}\psi^{(L;\ \Lambda)}) = -\,\Omega_{\Lambda L} = -\int d\tau\bar{u}_{\Lambda}\Omega u_{L}.
\end{aligned} \tag{3.14}$$

In this proof one has to make repeated use of the identities (3.13). For operators of the form (2.21) we may thus use the Dirac functions u_L and u_Λ to compute the matrix elements for states (L), (Λ), (O), and $(L;\Lambda)$. In general $\bar{\omega}\psi^{(L_1\cdots L_n;\ \Lambda_1\cdots\Lambda_m)}$ gives us wave functions $\psi^{(L_1\cdots L_{n-1}L';\ \Lambda_1\cdots\Lambda_m)}$, $\psi^{(L_1\cdots L_n;\ \Lambda_1\cdots\Lambda_{m-1}\Lambda')}$, $\psi^{(L_1\cdots L_nL';\ \Lambda_1\cdots\Lambda_m\Lambda')}$, and $\psi^{(L_1\cdots L_{n-1};\ \Lambda_1\cdots\Lambda_{m-1})}$. In constructing the corresponding matrix elements of $\bar{\omega}$, the antisymmetry of the ψ's in all L and all Λ must be considered.

Let us now consider transitions induced in the system by perturbations. For the present argument it does not matter whether these perturbations arise from the coupling of the system with another system, such as the radiation field, or by a direct introduction of time dependent forces. In either case, to a transition of the system in which the energy of the system changes by ΔE there will correspond a perturbing term in the Hamiltonian of the system which is of the form (2.5), and is given by the matrix $r_{(r)(s)}\exp(i\Delta Et/\hbar)$. The perturbation theory then tells us, that, to the first orders in v, and with suitable normalization, the transition probability $L\to L'$ induced by the perturbation will be

$$P_{(L)\to(L')}=(1/\hbar^2)\,|\,(L'|v|L)\,|^2 \qquad (3.15)$$

with $E^{(L')}-E^{(L)}$ necessarily equal to ΔE. Similarly

$$P_{(O)\to(L;\ \Lambda)}=(1/\hbar^2)\,|\,(L\Lambda|v|O)\,|^2;$$
$$E^{(L;\ \Lambda)}-E^{(O)}=\Delta E.$$

From (3.14) we see that to this order the transition probabilities are given by

$$P_{(L)\to(L')}=(1/\hbar^2)\,|\,v_{L'L}\,|^2 \qquad (a);$$
$$P_{(\Lambda)\to(\Lambda')}=(1/\hbar^2)\,|\,v_{\Lambda\Lambda'}\,|^2 \qquad (b); \qquad (3.16)$$

$$P_{(O)\to(L;\ \Lambda)}=(1/\hbar^2)\,|\,v_{L\Lambda}\,|^2 \qquad (a);$$
$$P_{(L;\ \Lambda)\to O}=(1/\hbar^2)\,|\,v_{\Lambda L}\,|^2 \qquad (b). \qquad (3.17)$$

This result shows that in the problems where the Dirac theory may be applied to determine the stationary states of the system, the first order transition probabilities, which give the probabilities of radiative and collision transitions, may be computed with the help of the Dirac wave functions $u_{(L)}$ just as in nonrelativistic theory. For these cases (3.16b) also shows that analogous methods may be applied to positives, and gives (3.17a) for the probability of pair production; it is this formula which is applied in computing the production of pairs by gamma-rays and electrons.

This result cannot be generally extended to the calculation of higher order transition probabilities, as is at once clear when we remember that a system which, when unperturbed, may very well admit the application of the Dirac equation, may, under the influence of the perturbation, cease to have this property. Thus the free electron or the electron in a Coulomb field may as is well known be treated by the Dirac equation; but when this system is coupled to the radiation field, the catastrophic transitions to negative energy states occur, and we cannot expect that a theory in which these transitions cannot be unambiguously excluded will give correct results. One can show, however, that to the second order in v, for any perturbation whatever, the direct use of the wave functions u_L, u_Λ and their corresponding energy values E_L, E_Λ gives, upon application of the usual perturbation methods, the correct result. For the proof of this the antisymmetry of the $\psi^{(L\cdots;\ \Lambda\cdots)}$ in the L's and Λ's is essential.

This result is sufficiently general to validate all the calculations made on the basis of the Dirac equation which have been compared with experiment. In particular it furnishes a generalization of Dirac's proof that "filling the states of negative energy" would not affect the scattering formulae of Thomson and Klein-Nishina. As far as perturbations induced by coupling an electron with the radiation field are concerned, calculations of transition to orders higher than the second cannot in any case be made, because in these higher orders the difficulties of the magnetic proper energy make an insurmountable ambiguity. For this magnetic proper energy the present theory, of course, gives new results; but these in no way remove the familiar difficulties of the radiation theory.

The limitation on the nature of the system which makes possible an unambiguous application of Dirac's equation, and which has played so large a part in the arguments of this section, admits a very simple interpretation. For this limitation means only that in such systems no genuine transitions to states of negative energy should, on the Dirac theory, be possible; on the

present theory these are the systems in which, though pairs may be present, their presence may be unambiguously detected only by disturbing the system with high energy radiations. To problems in which pairs are constantly being created by the fields present, the Dirac equation in its original form cannot be simply applied.

Throughout this section we have consistently neglected the interaction of the particles with each other, and supposed them to be affected only by given external fields. When these interactions are considered, the proofs which we have given of the applicability of the Dirac equation no longer hold, for in the original Dirac theory these interactions have of course no analogue. In particular, when these interactions are considered, one can no longer establish the rigorous validity of the scattering formulae, and the energy levels of electrons in static fields cease to be given correctly by the Dirac equation. The effects of these interactions are in general small and cannot, as we have seen, be unambiguously formulated in all cases. But within the limits here discussed, which from a physical point of view seem inevitable, we have shown that the Dirac equation, wherever it can consistently be applied, leads to the same results as the present theory. For such problems, and within these limits, it is thus not in general necessary to use the wave functions $\psi_{N,M}(r;\rho)$ at all; and this is fortunate, since the wave equations which determine them are in general intractable.

4.

In this section we have to consider some of the applications of the theory which have no analogue in the Dirac theory of the electron in its original form. We shall make no attempt to treat these exhaustively, but try only in a few simple cases to see how the results may be interpreted, and in what way the application of the theory is limited.

Let us look first at $E^{(0)}$. In a system in which directly observable pairs are not being created and in which therefore the solutions of Dirac's equations may be classified as L or Λ, there exists as we have seen a unique normal state for the pairs; the energy of this state is $E^{(0)}$ and is given by (3.11). This is the energy of these "nascent" pairs in the given field; when the field vanishes $E^{(0)}\to 0$; when the field does not vanish $E^{(0)}$ is in general negative. To establish the field, therefore, we do not in general have to do quite as much work as electromagnetic theory predicts; for in addition to the electromagnetic energy

$$E_e=(1/8\pi)\int d\tau(\mathfrak{E}^2+\mathfrak{H}^2) \tag{4.1}$$

in the electromagnetic field, there is an energy $E^{(0)}$ in the pairs formed by the field. This is in no sense a discrepancy with Maxwell's theory, according to which E_e should give the energy in empty space of the field $\mathfrak{E}$, $\mathfrak{H}$; for, as we have seen, the field will not be empty. This then is the simplest general interpretation of $E^{(0)}$.

If the field is itself produced by a charged particle—and here we shall assume that this particle is not an electron, but is so heavy that the reaction of the pairs upon it may be neglected—then the corresponding energy $E^{(0)}$ must be interpreted as proper energy of the particle. The only way in which $E^{(0)}$ here could be directly observable is by the creation or destruction of the particle; it could be inferred from the mass of the particle if the specifically electromagnetic energy of the particle could be computed in a satisfactory manner. For a point charge with a Coulomb field this proper energy $E^{(0)}$ is infinite and negative. It does not appear to be possible to interpret the difference between the electromagnetic and the pair energy of the particle in such a way that this difference should correspond to a finite mass; and in general it may be said that the present theory, which presupposes the particulate nature of the electron and positive, throws absolutely no light on their ultimate stability.

When more than one heavy charged particle produces the field, the energy $E^{(0)}$ will depend upon their relative positions. This dependence of the energy on the positions of the particles will correspond to a new term in the potential between them: i.e., to new forces. These forces will not in general be Coulomb forces and the deviations may be computed, and should ultimately be observable.

To make these considerations quantitative, let us consider the case of an electrostatic field, for which

$$\mathbf{A}=0; \qquad \mathfrak{E}=-\operatorname{grad} V. \tag{4.2}$$

When the field strength is not too great, we should

expect to be able to solve (2.33) and compute $\psi^{(0)}$, by perpurbation methods, and use (3.3) for $E^{(0)}$. A sufficient condition on the field strength is that it should always be small compared to m^2c^3/eh. In this case it will be permissible to neglect entirely the even part of V which can give effects only in higher order and corresponds to the nonuniform motion of the pairs when they are accelerated by the field. Then we find

$$E^{(0)} = -\sum_{r,\ \rho} |V_{r\rho}|^2/(E_r - E_\rho), \tag{4.3}$$

where now r, ρ index the positive and negative states of a free electron. The energy density in the pairs is of the order α compared to the electromagnetic energy. For the case that the potential varies relatively little in regions of the order of the Compton wave-length we find for the ratio of the two energy densities:

$$E^{(0)}/E_e = -\alpha\kappa. \tag{4.4}$$

Here κ is given by the integral

$$\kappa = (2/\pi)\int_0^\infty k^2(1+k^2)^{-5/2}dk + (4/3\pi)\int_0^\infty k^4(1+k^2)^{-5/2}dk, \tag{4.5}$$

where in turn k is $(1/2mc)$ times the magnitude of the vector difference of the momenta of the two particles of a pair. The second integral diverges for large values of k: the contribution of pairs of large momentum does not fall off rapidly enough to give a determinate result. This divergence does not depend upon the special simplifications introduced into the problem and arises from a genuine limitation of the present theory.

This limitation is of the same kind as that met in all applications of the quantum mechanical formalism to wave fields, and comes in the divergence of κ to a similar expression; it has its analogue in the now familiar divergence difficulties of dispersion theory which may be schematically formulated as the failure of such theories when applied to extremely small lengths or intervals of time. The difficulty here has nothing to do with the introduction of point singularities in the field; the electromagnetic field we are studying is slowly varying and nowhere infinite. An analogous failure of field theory in a problem in which no singularities occur is known in gravitational theory, where the quantum theoretic calculation of the gravitational energy of an electromagnetic wave gives a divergent result.[10] These difficulties thus tend to appear in all problems in which extremely small lengths are involved; the critical lengths are presumably of the order of the cla s-sical electron radius. The difficulties are of such a character that they are apparently not to be overcome merely by modifying the electromagnetic field of an electron within these small distances, but require here a more profound change in our notions of space and time, on which ultimately the quantum mechanical methods rest, and which in turn require the existence of stable particles for their definition. At present neither the precise point at which the theory breaks down nor the nature of the needed modifications can be determined; but it is clear that the sort of thing we must do with expressions like (4.5) is to break off the integral as it stands at some upper limit K corresponding roughly to the critical length e^2/mc^2, and trust a future theory to show that the contributions from k values greater than this are small. It is from this at once apparent that the theory in its present form can make no predictions whatever about the fields within the critical distance e^2/mc^2 of a charge.

In the case of the production of pairs, there is direct evidence that the present theory gives too large a probability for high energy pairs. For the probability of production of pairs by a beam of gamma-rays in the field of nuclei, when computed on the basis of the present theory, turns out to be much larger than any value which could be reconciled with the known penetrating power of high energy gamma-rays. And in this case too the failure of the theory seems in no way connected with the magnitude of the nuclear field in the immediate neighborhood of the nuclei. The fact that the theory should fail here is not very surprising when we remember that even on classical theory the model of a point electron, which underlies the present theory, would give altogether wrong results for the reaction of the electron to light of wave-length appreciably shorter than the critical length e^2/mc^2.

With this understanding, then, κ is given by

[10] L. Rosenfeld, Zeits. f. Physik **65**, 589 (1930).

$(4/3\pi)\ln 2K - 10/9\pi$ and is about 2 with $K \sim 1/\alpha$. The result then tells us that the work we must do to establish an electrostatic field is about 2 percent less than the energy stored in the electromagnetic field; the difference is supplied by the pairs. One might at first suppose that this result would have catastrophic consequences for electromagnetic theory. But one sees at once that the forces exerted upon a point charge (by a field which still varies little within λ_0), which serve to define the field strengths in terms of the unit of charge, are reduced by the pairs formed about the charge in just the ratio (4.4). And we may readily see that the forces exerted by two charges upon each other (again at distances large compared to λ_0), which serve to define the unit of charge, are similarly reduced; so that as long as we do not consider lengths smaller than λ_0—which vanishes as h goes to zero as the correspondence principle requires—the consistency of classical electromagnetic theory is not affected at all, and the difference between the "true" charges, and "true" fields and those which include the effect of the pairs, are not, under these circumstances, observable.[10a]

We may calculate simply this dependence of pair energy on the separation of two charges. If the charge on the particles is not greater than Ze, then the condition $|\mathfrak{E}| < m^2c^3/he$ means only that the particles shall stay far apart compared to $(\alpha Z)^{\frac{1}{2}}\lambda_0$. If the two particles are separated by a distance R, then

$$E_{\text{int.}}^{(0)} = -(\alpha e_1 e_2/4\pi^4\lambda_0)\int\int d\mathbf{k}d\mathbf{k}' \frac{[\epsilon\epsilon' - (\mathbf{k}\cdot\mathbf{k}') - 1]e^{i(\mathbf{k}-\mathbf{k}')\cdot R/\lambda_0}}{\epsilon\epsilon'(\epsilon+\epsilon')|k-k'|^4} \tag{4.6}$$

with $\mathbf{k}$, $\mathbf{k}' = (2\pi/mc)$ times the momentum of the positive and electron, and $\epsilon = (1+k^2)^{\frac{1}{2}}$. When $R \gg \lambda_0$, this may be evaluated and gives just

$$E_{\text{int.}}^{(0)} = -\alpha\kappa e_1 e_2/R. \tag{4.7}$$

This result shows that if we define the unit charge[11] in the usual way and the field strengths in terms of this charge, then the energy in the field will be given by E_e of electromagnetic theory. In situations where the charges are not strongly accelerated and in which no lengths small compared to λ_0 enter, the difference between the charge so defined and the true charge Ze, will not be observable.[12] In principle a detection of this difference between true and apparent charge could be expected for a violently accelerated electron; the effects would be of the order α for accelerations of the order mc^3/h. But for such accelerations the electron theoretic behavior of an electron is uncertain by effects of just the order α which is the ratio of the acceleration to mc^4/e^2. Although, therefore, the effects here discussed would in principle be capable of observation if we had an adequate electromagnetic theory with which to compare them and would in that case have to be taken into account in describing the behavior of the electron, they would appear in fact in the present state of the theory not to be unambiguously detectable by observation.

A similar argument shows that the deviations from (4.7) given by (4.6) for the field at distances $R < \lambda_0$ from an electron are also in this sense unobservable, that they are masked by the

[10a] *Note added in proof:* Because of the gauge dependence of the theory no unambiguous calculation of the polarization energy of the pairs in a magnetostatic field can be made; and in fact no straightforward choice of gauge gives for this energy that value formally equal to (4.4), which alone is consistent with relativity. This consideration shows that, quite apart from the ambiguities discussed above, the present calculations of the reaction of the pairs on the field may have to be essentially modified.

[11] Through the kindness of Professor Lawrence, we have just seen Dirac's report to the Solvay Congress. In this report, Dirac, from a different point of view, is led to a result that the charge of an electron defined in the usual way is not the true charge. The expression which Dirac derives for the difference between these charges is in complete agreement with (4.7).

[12] This can be said in another way: Because of the polarizability of the nascent pairs, the dielectric constant of space into which no matter has been introduced differs from that of truly empty space. For fields which are neither too strong nor too rapidly varying the dielectric constant of a vacuum then has the constant value $\sim(1+\kappa\alpha)$. Because it is in practice impossible not to have pairs present, we may redefine all dielectric constants, as is customarily done, by taking that of a vacuum to be unity.

electron theoretic uncertainties in the behavior of an accelerated electron. Thus these deviations in the field give a shift in the energy levels of an atomic electron predicted by the Dirac theory; but these shifts are never greater than the uncertainties in the energy levels which arise from our ignorance of the reaction of the electron to its own radiation field.

For protons, for which we may perhaps suppose that the electron theoretic ambiguities arise only for the far larger accelerations Mc^4/e^2 the deviations from the Coulomb law should in principle be detectable. These deviations arise at distances $R\sim\lambda_0$. For $R\ll\lambda_0$, but still, in order that KR may be regarded as large, large compared to e^2/nc^2, (4.6) gives:

$$E_{\text{int.}}^{(0)} = -(\alpha e^2/R)[\kappa+((10/9)\pi) +((4/3)\pi)(\ln(2\pi R/\lambda_0)+\gamma)] \quad (4.8)$$

with γ = Euler's constant. This corresponds to an increase of effective charge of the proton of

$$\delta e\sim(2\alpha e/3\pi)\ln(\lambda_0/25R). \quad (4.9)$$

The effect is small, but may perhaps be detected by scattering experiments. For the scattering of protons by protons, deviations from the scattering for a Coulomb field of the order of one percent should appear for proton energies of a hundred thousand volts. It must again be emphasized that for very close distances of approach, where these deviations may be large, the theory may not be applied; for the methods here used to give a meaning to κ must rather be thought of as the first steps in an approximation based upon the actual smallness of α.

The Fine Structure of the Balmer Lines

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A new method of treating interferometer patterns of doublets has been applied to the study of the Balmer series. The method involves the measurement of the intensities of the minima between the members of the doublet and between successive orders of interference. This gives for the separations of the centers of gravity for Hβ, $\Delta\nu=0.3298\pm0.0004$; for Hγ, $\Delta\nu=0.3388\pm0.0004$; for Hδ, $\Delta\nu=0.3451\pm0.0006$; and for Hε, $\Delta\nu=0.3506\pm0.0006$. These values are far too small to agree with the ordinary theory of these lines and it must be concluded that the structure is not quite that expected. The discrepancy may lie in the neglect of the radiation reaction in the calculation of the energy levels.

BECAUSE the present quantum theory permits all of the features of the hydrogen spectrum to be calculated without any approximations of a mathematical nature, it is of interest to study this spectrum with especial care. Previous work on the fine structure of the Balmer series has led to the conclusion that the positions of the individual lines are those given by the Sommerfeld formula, when the selection rules derived on the basis of the electron spin idea are used.[1] Since the Dirac theory of this spectrum leads to the Sommerfeld formula for the positions, and to selection rules which are analogous to those of the alkalis, the observations have been taken as lending strong support to the validity of Dirac's relativistic equation. However, in spite of the apparently satisfactory way in which the observed positions have agreed with those to be expected, there has usually been some discrepancy between the observed and calculated relative intensities of the two members of each doublet. This has usually been ignored since it has been felt that the knowledge of the conditions of excitation is insufficient to give an accurate prediction of the intensities.

Because of the satisfactory way in which the apparent separations of the hydrogen doublets, i.e., the separations of the centers of gravity of the two component groups, seemed to fit the theory, we hoped to use this separation as a means of experimentally determining the fine structure constant, which has been the subject of so much recent discussion. This determination would require the measurement of this apparent separation for the higher members of the series, since for the higher members the separation of the centers of gravity approaches the separation of the lower states. To obtain the desired accuracy in the fine structure constant it would be necessary to measure this apparent separation with a precision of the order of 0.1 percent. The first two members of the series are not very suitable for this purpose, because the two component groups, which are observed, are composed of several lines whose separation is a very considerable fraction of the apparent doublet separation; but since the theory must be assumed to connect the fine structure pattern with the fine structure constant, it can be assumed also to make the suitable corrections for the first two lines.

It has been found possible to approach the required precision on the first five members of the series, but the results are so far from those to be expected that it is clear we are not measuring the fine structure constant, but have attained to that degree of precision in which the theory is no longer satisfactory. One possible explanation of this is that the effect of the interaction between the radiation field and the atom has been neglected in computing the frequencies.

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[1] G. Hansen, Ann. d. Physik **78**, 558 (1925). W. V. Houston, Astrophys. J. **64**, 81 (1926). Sommerfeld and Unsöld, Zeits. f. Physik **36**, 259 (1926); **38**, 237 (1926). Kent, Taylor and Pearson, Phys. Rev. **30**, 266 (1927).

The results presented here give an indication of the amount by which the theory is in error, but because of the complexity of the patterns, they do not give a great deal of information as to the exact nature of the discrepancy.

Description of the Apparatus

The source of light is a simple discharge tube, 2 cm in diameter and about 50 cm long. It is bent into the form of a U so that the central part can be immersed in a thermos bottle full of liquid air. The window is so arranged that only the part of the tube which is under the liquid air can be seen from the slit of the spectrograph. The excitation is by means of a direct-current generator.

The hydrogen is prepared by electrolysis, and is admitted in small quantities to the discharge tube through which helium at a pressure of about 0.5 mm is kept circulating. The circulating system contains two traps immersed in liquid air, one of which is filled with charcoal. After the system has been sealed up for several weeks, most of the impurities are absorbed on the charcoal, and the spectrum in the center of the discharge tube consists almost entirely of the Balmer series. The stronger helium lines appear in the long exposures. The structures seem to come out most clearly when the pressure is not over 0.5 mm and the current is not over 0.1 amp. These conditions are not very sharply defined, however, and we hope later to investigate more thoroughly the effect of the conditions of excitation.

The spectrographic apparatus is the usual combination of a Fabry-Perot interferometer with a prism spectrograph. The camera has a focal length of 80 cm. This is about twice that of the collimator. The magnification of the slit image, which is obtained in this way, helps to eliminate the effect of the photographic plate grain.

The interferometer was loaned us by Dr. Babcock of the Mt. Wilson observatory, and we wish to express here our appreciation of his kindness in permitting us to use this magnificent instrument. The plates are about 11 cm in diameter, and since the diameter of the collimator lens is only 3 cm, the best part of the plate surfaces can be selected for use. The plates are silvered frequently, by evaporation,[2] with enough silver to give about 92 percent reflection. The transmission is then about 4 percent, and the resolving power seems to be the maximum consistent with sufficient light intensity. The exposure times range between 2 and 5 minutes, so that no especial precautions are taken to control the temperature of the spectrograph.

The method used for measuring the apparent doublet separation requires the determination of relative intensities. For this purpose intensity marks are put on each plate by the arrangement shown in Fig. 1.[3] A 500 watt lamp S is enclosed in a box with an opal glass window G. This provides a fairly uniform source of light about 9.5 cm square. By means of a lens L_1, this source is focussed upon the cylindrical lens C through the 90° prism P. At R is a grating of the form shown in 1B. The longest slot is 48 mm and the shortest is 0.5 mm long. There are twelve slots in all and they are each 2 mm wide. This grating is focussed on the slit by the cylindrical lens C whose axis is horizontal. The lens L_2 focusses C upon the collimator lens L_3.

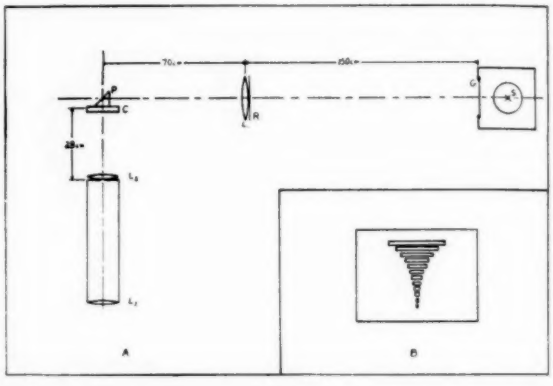

Fig. 1. (A) The arrangement of the apparatus for putting density marks on each plate. (B) The form of the screen for producing the density marks.

[2] The silver is applied with the apparatus developed by Dr. J. D. Strong and described in Rev. Sci. Inst. **2**, 189 (1931). We are much indebted to him for his assistance in this work.

[3] This method has been described by G. Hansen, Zeits. f. Physik **29**, 356 (1926).

When properly adjusted, this arrangement gives a series of illuminated strips whose intensity in the center is proportional to the length of the corresponding slot in R. Although no elaborate tests have been made to check the intensity ratios, they appear to have the expected values.

Method of Measurement

It is probably impossible to attain the required precision by the ordinary methods of measurement. The two component groups in each series member are so wide that the uncertainty in the location of the maximum, or the center of gravity, introduces more error than can be permitted. Hence it is necessary to make the measurements by analyzing the intensity distribution.

There are three principal reasons for the breadth of the groups in the Balmer series. In the first place each of the individual lines has a certain inherent width due to the radiation damping. In the second place, the observations are made on the two component groups of lines which constitute the doublet. Each of these groups consists of either two or three individual lines, and so the whole group has a width which is at least as great as the separation of these individual lines. The third and predominant cause, however, is the Doppler effect, due to the temperature motion of the atoms. It has been assumed throughout that all causes of broadening, except the Doppler effect and the complex nature of the groups, can be neglected.

The shape of a single line which is broadened by the Doppler effect is[4]

$$I = I_0 e^{-\alpha(\delta\lambda)^2}, \quad \text{with} \quad \alpha = mc^2/2kT\lambda^2. \qquad (1)$$

I is the intensity of the line at the wave-length $\lambda \pm \delta\lambda$, I_0 is the intensity of the line at the center, c is the velocity of light, k is the Boltzmann constant, T is the absolute temperature, and m is the mass of the atom or molecule which emits the line. If $\Delta\lambda$ and $\Delta\nu$ represent the half width of such a line at half maximum intensity, measured in wave-length and wave number, respectively, then

$$\Delta\lambda/\lambda = \Delta\nu/\nu = 3.57 \cdot 10^{-7}(T/M)^{\frac{1}{2}}. \qquad (2)$$

In this equation, M is the molecular weight of the atom or molecule which emits the line. It is to reduce as far as possible the broadening of this kind that the discharge tube is operated under liquid air, but even under these conditions, the individual lines have a considerable width. If a temperature of 100°K is assumed, the calculated half widths range from 0.05 cm^{-1} for the individual lines of Hα to 0.09 cm^{-1} for those of Hϵ. The apparent doublet separation which is to be measured is only two to three times as large as the minimum separation that could be observed with individual components as wide as this.

Because of these facts it is impossible, at least at the available temperature, to resolve the component groups, of a doublet, into the individual lines of which they are composed. It is only possible to compute the theoretical shape and to determine experimentally the distribution of intensity. However, instead of trying to determine a large number of points on the curve of intensity as a function of wave-length, we have for the present concentrated on the determination of the positions of the centers of gravity of each of the component groups. Other characteristic points would serve as well, but the subsequent analysis shows that the center of gravity is a point which is easily located. If it is assumed that the relative intensitives and the positions of the individual lines are those required by the theory of Dirac, the scale of the pattern can be determined by determining the separation of the centers of gravity of the component groups. This can be done with considerable precision by the methods indicated below. If, however, this assumption is not made, the measurements we have made give a somewhat less accurate value for some sort of an apparent doublet separation.

The ordinary method of measuring an interferometer pattern requires the estimation of the location of some characteristic point on each fringe, usually the center of gravity, either on the original plate or on a microphotometer curve. From the location of these points the diameters of the corresponding circles can be determined, and these give the value of the fractional order of interference at the center of the pattern for each line. In terms of these orders of inter-

[4] Rayleigh, Phil. Mag. **29**, 274 (1915).

ference, the wave-number difference between the two lines is

$$\Delta\nu = \Delta p/2d, \tag{3}$$

where Δp is the difference between the orders of interference of the two lines and d is the separation between the surfaces of the interferometer plates.[5] The idea of our procedure is to find the plate separation for which $\Delta p = 0.500$, without actually measuring the fractional orders of interference. If the pattern is due to a pair of single lines, each of which is symmetrical about its center, the pattern will then consist of a series of fringes separated by minima which are all of the same intensity. When the difference between the orders of interference is greater or less than 0.500, the minima between two fringes with the same integral order of interference will be less intense or more intense, respectively, than the minima between fringes of successive orders of interference. This behavior of the minima provides a sensitive test for the satisfaction of this condition. The method, then, consists in photographing the interference pattern with different values of d and selecting that value which satisfies this test.

Since it is not practical to construct a series of interferometer separators in which the thicknesses differ by much less than 0.05 mm, it is necessary to find a reliable method of interpolation between the separations used. This requires the definition of a measure of the lack of equality between the intensities of the two kinds of minima. This measure should be somewhat independent of the widths of the fringes, so as not to be too much affected by changes in the exposure time and in the accuracy of the adjustment of the interferometer. These conditions are satisfactorily met by the following analysis.

If the spectrum consists of two single lines, both symmetrically broadened by the Doppler effect, the intensity distribution in the interference pattern is given by

$$I(x) = \sum_{n=-\infty}^{\infty} e^{-\beta(x-n)^2} + A \sum_{n=-\infty}^{\infty} e^{-\beta(x-h-n)^2}. \tag{4}$$

In this expression x is the distance, measured in orders of interference, from one of the maxima which is taken as the origin. h is the wave-number difference between the two spectral lines divided by the spectral range of the interferometer pattern. β is the constant which gives the width of the individual lines. Because of the units in which x is measured, β is equal to the α of Eq. (2) multiplied by the square of the spectral range of the pattern. In making the approximations it will be assumed that $\beta > 16$. This condition is satisfied for the hydrogen lines with the interferometer separations used. A is the ratio of the intensities of the two lines which compose the pattern, and for convenience the origin is taken so that A is not greater than one.

From Eq. (4) it can be shown that the positions of the first two minima on the positive side of the origin are given by

$$\begin{aligned} x_1 &= (1/8+\epsilon/4)\{1+(8A-\beta)/(8-\beta)\} \\ \text{and} \\ x_2 &= 1-(1/8-\epsilon/4)\{1+(8A-\beta)/(8-\beta)\}, \end{aligned} \tag{5}$$

where $\epsilon = h - 0.500$. ϵ is a measure of the amount by which the fringes due to the line of intensity A are distant from the position halfway between the adjacent fringes due to the other line. Because of the assumed lower limit for β it is unnecessary to use more than one term in each of the sums in Eq. (4) to determine the intensities at these minima. Let M_1 be the intensity at x_1, let M_2 be the intensity at x_2, and let $(8A-\beta)/(8-\beta) = 1+S$. Then, with sufficient accuracy

$$\begin{aligned} M_1 &= e^{-\beta(\frac{1}{2}+\epsilon/2)^2}\{e^{-\beta(\frac{1}{2}+\epsilon/2)^2 S} + A e^{\beta(\frac{1}{2}+\epsilon/2)^2 S}\}, \\ M_2 &= e^{-\beta(\frac{1}{2}-\epsilon/2)^2}\{e^{-\beta(\frac{1}{2}-\epsilon/2)^2 S} + A e^{\beta(\frac{1}{2}-\epsilon/2)^2 S}\}. \end{aligned} \tag{6}$$

Since S and ϵ are both small compared with unity, this gives

$$\log M_1/M_2 = -\beta\epsilon/2. \tag{7}$$

If the quantity β were strictly constant, and did not vary at all from one exposure to the next, Eq. (7) would be sufficient to determine the variation of ϵ with the interferometer separation. It would only be necessary to plot $\log (M_1/M_2)$ against the separation, d, when the slope would give the value of β, while the intercept would be the value of d for which $\Delta p = 0.500$. However,

[5] W. V. Houston, Astrophys. J. **64**, 81 (1926).

β does vary a little from one exposure to the next and so it is desirable to evaluate it for each plate. This can be done from the relative intensities of the maxima and the minima. For small ϵ and small S, Eq. (6) gives

$$M_1+M_2=2(1+A)e^{-\beta/16}. \tag{8}$$

Then, if L_1 and L_2 are the intensities of the two maxima, Eq. (4) gives

$$\begin{aligned} L_1+L_2&=(1+A)\{1+2e^{-\beta(\frac{1}{4}+\epsilon)^2}\} \\ &=(1+A)(1+2e^{-\beta/4}). \end{aligned} \tag{9}$$

From these it follows that

$$\frac{L_1+L_2}{M_1+M_2}=\frac{1+2e^{-\beta/4}}{2e^{-\beta/16}}. \tag{10}$$

Thus, from a knowledge of the ratio in (10), the quantity β can be determined. With this, ϵ can be determined from (7).

Symmetry of the Component Groups in Each Doublet

Since the method which has just been described is based on the assumption that the two spectral lines, whose wave-number separation it is desired to measure, have the shape given by Eq. (2), it is necessary to investigate the extent to which this is true of the component groups in the Balmer series. If two individual lines of intensities 1 and A are separated by the distance t, the distribution of intensity due to the two of them is

$$\begin{aligned} I&=\exp[-\alpha z^2]+A\exp[-\alpha(z-t)^2] \\ &=\exp[-\alpha(z-g)^2]\left\{\exp\left[-\frac{\alpha t^2A}{1+A}\left(\frac{2z}{t}-\frac{A}{1+A}\right)\right]+A\exp\left[\frac{\alpha t^2}{1+A}\left(\frac{2z}{t}-\frac{1+2A}{1+A}\right)\right]\right\}, \end{aligned} \tag{11}$$

where $g=At/(1+A)$. If the separation of these lines is much less than their half width, i.e., if $\alpha t^2\ll 1$, this intensity distribution is very similar to that of a single line at the point $z=g$, and with the intensity $1+A$. For large enough values of z, of course, this will be a poor representation, but for these values of z the whole intensity will be so low that the discrepancy will not be important. For the component groups in the Balmer series, the theoretical positions of the individual lines are such that this is a good representation for all except Hα and Hβ. These must be separately considered.

Although Eq. (11) shows that the general shape of the composite group of two lines is well given by an exponential curve, it is not well adapted to showing the extent to which the group is symmetrical about its center of gravity. If it is to be perfectly symmetrical, it is necessary that all of the odd powers of z, in a power series expansion of the bracket, shall vanish. This they obviously do not do. It is sufficient, however, for the application of the method described above, that these odd powers should be negligible in the regions in which the intensity of the minima are measured. The determination of the conditions under which this amount of symmetry is present is more difficult from Eq. (11) than from an expansion in terms of the orthogonal functions of the Hermite polynomials. If a group consists of a set of individual lines of intensity A_i, whose centers are located at the points h_i, the intensity due to the whole group can be represented as follows. Let

$$\Phi_s=(2^s s!\sqrt{\pi})^{-\frac{1}{2}}H_s(z)e^{-z^2/2}; \tag{12}$$

then

$$\begin{aligned} \sum A_i e^{-\alpha(z-t_i)^2}&=\pi^{\frac{1}{4}}(2\alpha)^{-\frac{1}{2}}\sum_{s=0}^{\infty}(2^{s-2}s!)^{-\frac{1}{2}}(2\alpha)^{s/2}\sum_i A_i e^{-\alpha t_i^2/4}t_i^s\Phi_s(\xi) \\ &=(2\alpha)^{-\frac{1}{2}}e^{-\alpha z^2}\sum_{s=0}^{\infty}(2^{s-1}s!)^{-1}(2\alpha)^{s/2}H_s(\xi)\sum_i A_i e^{-\alpha t_i^2/4}t_i^s, \end{aligned} \tag{12a}$$

where $\xi=(2\alpha)^{\frac{1}{2}}z$.

In order that the expression (12a) shall have the required symmetry, it is necessary that the coefficients of the odd numbered polynomials be zero. The coefficient of H_1 can be made zero by adjusting the origin from which z and t_i are measured. The position so determined is just the center of gravity except that each individual line is given a weight equal to its intensity multiplied by $e^{-\alpha t_i^2/4}$ instead of simply equal to its intensity. This factor, however, makes a negligible difference since in the cases in which the required symmetry exists the factor is very close to unity.

The sufficiency of the symmetry then depends upon the magnitude of the coefficient of H_3 multiplied by the value of H_3 at the point where the minimum in the interference pattern occurs. The leading term in H_s is $(2^{\frac{1}{2}}\alpha^{\frac{1}{2}}z)^s$, and the coefficient is $(2\alpha)^{s/2}\sum_i A_i t_i^s$. Hence if the quantity $(4\alpha t_i z)$ is sufficiently small at the value of z for which the minimum occurs, the convergence of the series is rapid enough to neglect all of the odd polynomials of index greater than one. As an example, this criterion may be applied to the violet component group of $H\beta$. Table II gives the theoretical structure of this group. The individual lines are located so that $\Delta\nu/R\alpha^2$ has the values 0.06543 and 0.07324, measured from the position given by the simple Balmer formula. It is important to notice that the α in this table is the fine structure constant and has nothing to do with the α used above to give the width of the lines. The two uses can be easily distinguished by the context. The point at which the minimum will be located is roughly halfway between the two component groups or at 0.04240. The two intensities A_1 and A_2 are 7.81 and 1.535, while the values of t are $t_1=0.00128$ and $t_2=-0.00653$. The value of z which is of interest is $z=-0.02956$. The value of α, when these units are used for position, is about 2500. With these values, the term in the summation over s, in Eq. (12a), for which $s=0$ is 18.69. The term for which $s=1$ is zero because of the use of the center of gravity as the origin. The term for which $s=2$ is 1.51, while the term for which $s=3$ is 0.3. Thus the term in H_3 contributes about 1.6 percent of the intensity at this minimum when the quantity $(4\alpha t_i z)$ has the value 1.95. For $H\gamma$, where $(4\alpha t_i z)=0.98$, the contribution of the H_3 term is small enough to be neglected. A similar analysis can be applied to the other component group.

The result is, then, that for $H\gamma$ and the higher members of the series, the two component groups of which the doublet is composed are sufficiently symmetrical for the application of the method described above. It also appears that the groups in $H\beta$ are not symmetrical enough to give good results. To make sure of this point a curve was plotted of the intensity distribution to be expected from the theoretical structure of $H\beta$. By actually drawing the interference pattern to be expected it was shown that the method which we use will give the wave-length difference between the centers of gravity with an error less than the experimental uncertainty. A similar graphical treatment of $H\alpha$ showed that the results would require a correction of the order of one percent.

Measurement of the Plates

To apply the method described we have made a microphotometer curve of each interference pattern and of the intensity marks put on each plate. From this curve it is possible to read the intensity of each maximum and each minimum in terms of the intensity of the strongest intensity mark. Since the intensity of illumination is not quite uniform over the image, it is necessary to compare each minimum with the average of the two adjacent minima. This ratio, taken for each minimum and averaged over the whole plate, gives the quantity M_1/M_2. In a similar way the ratio $(L_1+L_2)/(M_1+M_2)$ is averaged over the whole plate. From the plate average of this quantity, the quantity β is determined by Eq. (10). Eq. (7) then gives the value of ϵ which is characteristic of the plate.

The quantity ϵ is then plotted as a function of the separation of the interferometer surfaces, d. The point at which this curve crosses the axis is then the value of d which is inserted in Eq. (3) to find the wave-number separation of the centers of gravity of the component groups. If each component group is strictly given by the exponential curve, these graphs should be straight lines. If, however, the groups have any form whatever which is sufficiently symmetrical,

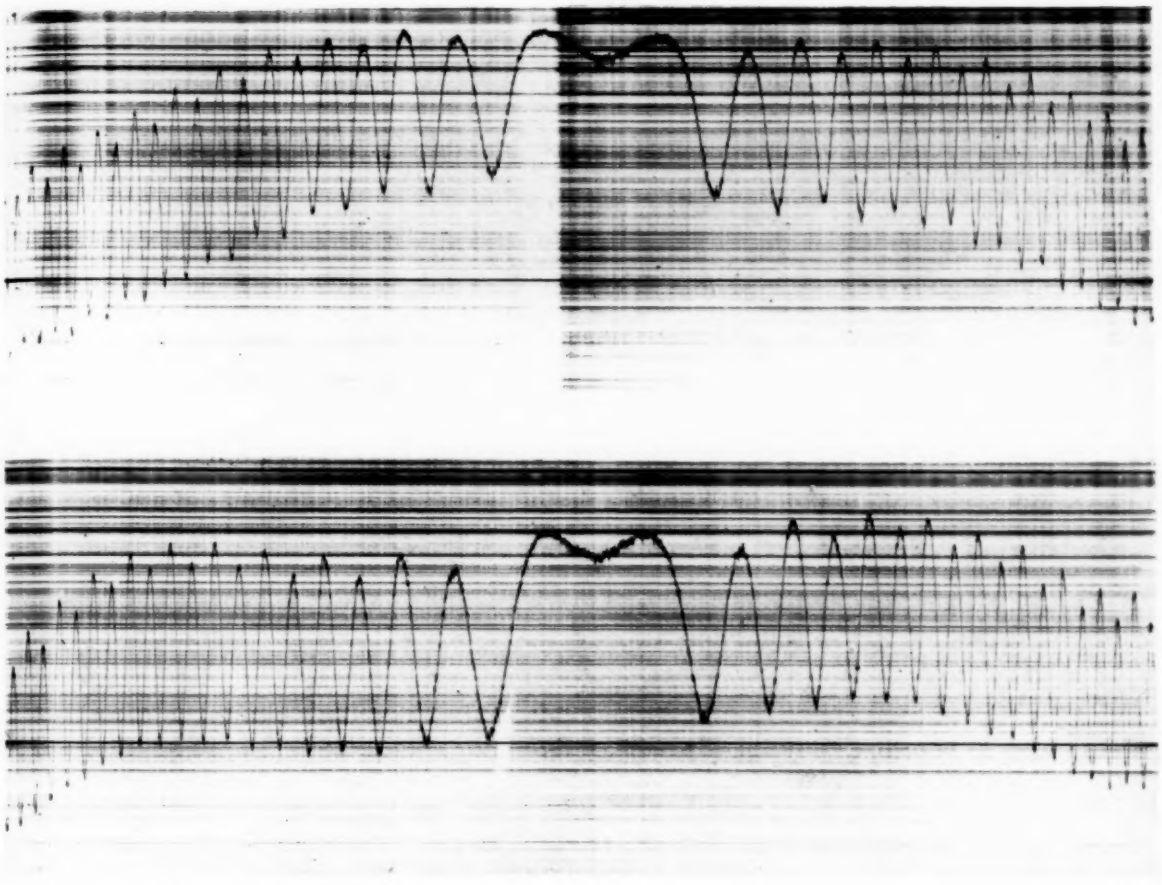

FIG. 2. (a) Microphotometer curve for the line Hβ at 7.499 mm separation. It can be seen from this curve that the more intense minimum has the stronger component inside of it and the weaker component outside of it. Hence $\Delta p < 0.50$. (b) Microphotometer curve for the separation 7.611 mm. The two minima are almost the same intensity, but a careful examination shows that $\Delta p > 0.50$.

the intersection of the curve with the axis will still give a value of d which will give the wave-number separation of the centers of symmetry of the groups. The detailed analysis of this fact has not been presented, but it can be easily seen from the general method. If the component groups are not symmetrical, the point at which the graph crosses the axis will still give the value of d for which the two minima are of the same intensity, and if the shape of the groups were known experimentally or theoretically, this datum would give their wave-number separation.

RESULTS

Fig. 2 shows a pair of microphotometer curves for Hβ. These show clearly the change in the interferometer pattern as the plate separation, d, is changed. It is possible to estimate from an inspection of these curves, the value of d at which $\Delta p = 0.500$. Fig. 3 shows the curves in which $(1/\beta) \log (M_1/M_2)$, in arbitrary units, is plotted against d. The vertical lines extending above and below the plotted points represent the mean deviation of the values from different plates. These are of the same order of magnitude, but are in general a little smaller, than the deviations within the individual plates. Hence the uncertainty is possibly greater than is indicated by these lines. Table I gives the measured values of the separations of the centers of gravity. The uncertainties in this table are merely estimated from the spread of the points in the curves.

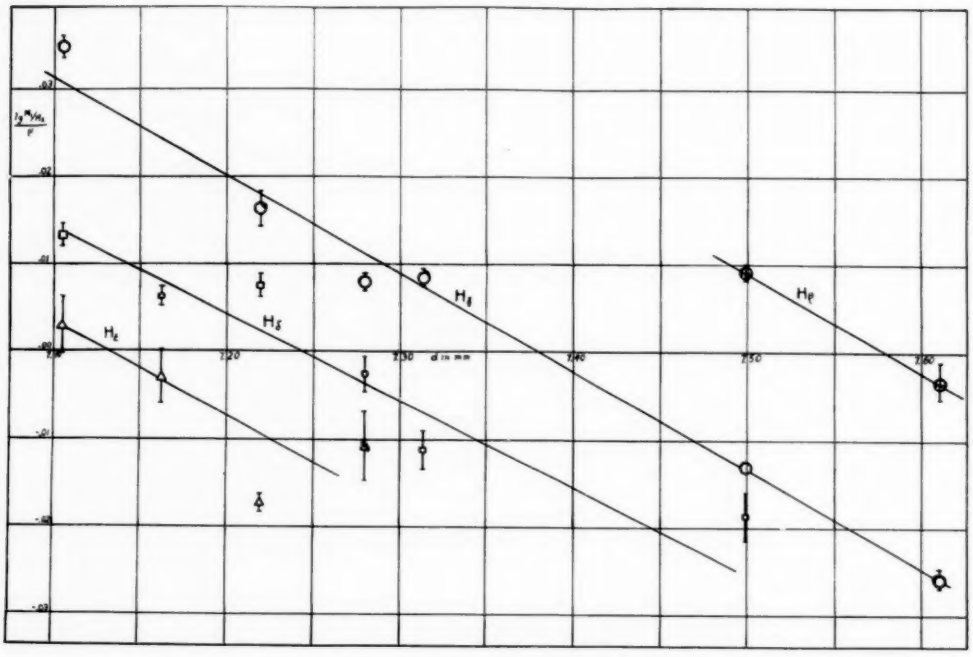

FIG. 3. Curves in which $(\log M_1/M_2)/\alpha$ is plotted against the interferometer separation in mm.

TABLE I.

Line	d_0 (mm)	$\Delta\nu$ (cm^{-1})
Hβ	7.58 $\pm$0.01	0.3298$\pm$0.0004
Hγ	7.38 $\pm$ .01	.3388$\pm$ .0004
Hδ	7.245$\pm$ .015	.3451$\pm$ .0006
Hϵ	7.13 $\pm$ .015	.3506$\pm$ .0006

The method of measurement we have used is not directly applicable to Hα, and so we have not given much attention to it. We have, however, made five measurements of three plates in the ordinary fashion, and have obtained $\Delta\nu = 0.3171 \pm 0.0020$. On the other hand, with an interferometer separation of 7.91 mm the minima were far from being equal. A rough estimate indicates that the minima would be equal at about 8.1 or 8.2 mm. If this could be applied directly to determine the separation of the centers of gravity of the lines, the separation would be 0.3086 or 0.3049. Of course this direct application is not justified in the case of Hα, but the graphical analysis based on the theoretical form of this line, indicated that these results need to be increased by only about one percent to get the correct separation. Hence we must conclude that the method of direct visual measurement gives results which are too large, at least for this line. This is not difficult to understand, since the peak of the line stands out much more on a plate than does the weaker companion, and so the measurement tends to be of the strong lines, rather than of the center of gravity. We hope to make a further analysis of Hα immediately, but this observation explains the difference between our present results and previous work on these doublets.

DISCUSSION OF THE RESULTS

Table II shows the position and the intensity of each of the components of the first five members of the Balmer series, as computed from the present theory, without the inclusion of the effect of a nuclear magnetic moment. The positions are those given by Sommerfeld's formula, which gives the energy levels permitted by Dirac's theory. The intensities are calculated by the methods of Sommerfeld and Unsöld.[1, 6] For small atomic numbers, this method gives the same results as a more rigorous calculation with relativistic functions.

[6] Saha and Banerji, Zeits. f. Physik **68**, 704 (1931), get the same results from Dirac's relativistic theory.

TABLE II. *Theoretically predicted structure of the Balmer lines.*

Component	Hα I	Hα $\Delta\nu/R\alpha^2$	Hβ I	Hβ $\Delta\nu/R\alpha^2$	Hγ I	Hγ $\Delta\nu/R\alpha^2$	Hδ I	Hδ $\Delta\nu/R\alpha^2$	Hε I	Hε $\Delta\nu/R\alpha^2$
$2s_{1/2}-np_{1/2}$	1.04		1.41		1.58		1.667		1.808	
		0.05035		0.06543		0.07132		0.07407		0.07552
$2p_{1/2}-ns_{1/2}$	0.097		0.125		0.137		0.143		0.146	
$2s_{1/2}-np_{3/2}$	2.08		2.81		3.16		3.333		3.616	
		.06887		.07324		.07532		.07639		.07698
$2p_{1/2}-nd_{3/2}$	5.00		5.00		5.00		5.00		5.00	
CG viol.	8.217	.06631	9.345	.07196	9.877	.07462	10.143	.07598	10.570	.07671
$2p_{3/2}-ns_{1/2}$	0.195	.01215	0.250	.00293	0.274	.00862	0.286	.01157	0.293	.01302
$2p_{3/2}-nd_{3/2}$	1.00	.00637	1.00	.01074	1.00	.01282	1.00	.01389	1.00	.01448
$2p_{3/2}-nd_{5/2}$	9.00	.01254	9.00	.01335	9.00	.01416	9.00	.01466	9.00	.01497
CG red	10.195	.01146	10.250	.01284	10.274	.01388	10.286	.01440	10.293	.01487
$\Delta CG/R\alpha^2$		.05485		.05912		.06074		.06158		.06184
$\Delta\nu$	1.243	.3191	1.098	.3440	1.040	.3534	1.014	.3583	0.974	.3598
Observed			1.089	.3298	1.041	.3388	1.009	.3451	1.006	.3506
Obs./Calc.				.959		.959		.963		.974

The first column gives the designation of the components in the notation for alkali spectra. The succeeding columns are devoted to the separate series members. The first column under each member gives the intensity, while the second column gives the displacement of the component from the position given by the Balmer formula, in terms of $\Delta\nu/R\alpha^2$. The corresponding quantities are given for the centers of gravity of the red and violet components. The fourth from the last row gives the separation of the centers of gravity in terms of $\Delta\nu/R\alpha^2$, while the third from the last row gives this separation in cm^{-1} when $R\alpha^2$ is taken as 5.818. This is the value given by Birge[7] as the best. In the same row, in the intensity columns, are given the ratios of the intensity of the red component to the violet component. The second from the last row gives the corresponding observed quantities, while the last row gives the ratio of the observed to the calculated separations. Hα is not included in these observed values as we have not yet made a careful measurement on it.

The predicted separations are in every case greater than those observed. The two sets could be brought together fairly well by using $(1/\alpha) = 139.9$. This change is so great, however, that it seems impossible that there should be any such error in the constants composing α, and we are forced to the conclusion that the theory, as we have used it, is inadequate to explain the observations.

One possible explanation of this discrepancy would be that the conditions of excitation are such that the relative intensities are not those assumed in Table II. It is impossible to absolutely exclude this possibility without some further investigation, but the fact that the observed intensity ratios of the component groups of the doublets are so near to the calculated values, gives some support to the belief that the intensities are all about as calculated.

Another possibility is that the effect of the nuclear magnetic moment is sufficient to so distort the structure that the centers of gravity have not the calculated separation. If it is assumed that the spin moment of the nucleus is $\frac{1}{2}$, and that the magnetic moment is that to be normally expected in view of the mass of the nucleus, the separation of the levels into which the $2s$ term is split is 0.002 cm^{-1},[8] while the separations of all other levels is much smaller. The relation of these two levels to the original is such that for transitions to this particular state, the center of gravity of the line is not displaced at all. For some of the other components there is a displacement of the center of gravity, but it is so small as to be entirely within the uncertainties of our measurement. Of course, if the g-factor of the nucleus were very much

[7] R. T. Birge, Phys. Rev. **40**, 207 (1932).

[8] G. Breit, Phys. Rev. **35**, 1447 (1931). H. Bethe, Handb. d. Physik **24**, 388.

larger than is to be expected there might be some effect.[9] On this account we have not included the effect of this hyperfine structure in Table II and believe we are justified in ignoring it.

Another explanation of these results has been pointed out to us by Professor Bohr and Professor Oppenheimer. The calculation of the energy levels has been made without the inclusion of the interaction between the electron and the radiation field. The customary procedure is to compute the levels in this way and then to introduce the interaction with the field as a perturbation. It is not known at the present time, how to combine the atom and the field into one system, but it is clear that when such a combination is made, a relative displacement of the levels may be expected, and it should be of the order of α times the fine structure separation. This is the order of magnitude of the effect we have observed.

The order of magnitude of the effect is also indicated by calculations of the natural width of the lines. the $2p$ level has the shortest lifetime of any of those involved. Its life is 0.16×10^{-8} sec.[10] By the uncertainty principle this gives a width of 0.02 cm^{-1}, which is larger than any of the observed discrepancies. Hence it seems to us very probable that this is the cause of the discrepancy we have observed.

[9] The value of the magnetic moment observed by Estermann, Frisch and Stern, Nature **132**, 169 (1933), is still much too small to produce any observable effect.

[10] H. Bethe, Handb. d. Physik **24**, 444.

The Paschen-Back Effect. I. *L-S* Coupling; the $^3P^3D$ Multiplets of Zn and Cd

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Darwin's calculations for Zeeman effect are applied to the Zn and Cd $^3P^3D$ multiplets. The frequencies and intensities thus calculated show very good agreement with the experimental values. A considerably smaller discrepancy between experiment and older quantum theory is noted for the Cd than for the Zn as should be expected because of the much larger multiplet separation in the case of the former.

THE new quantum mechanics has been applied to the calculation of standard Zeeman effect at all field strengths by Heisenberg and Jordan,[1] and Darwin.[2] Intensity rules have been given by Kronig[3] and Fowler.[4] Mensing[5] has applied these results to the cases of the $^2P^2D$ multiplet of Na and to the $^3P^3D$ multiplet of Mg. In these two cases the D separation is very much smaller than the separation of the P levels so that a very strong field for the D levels constitutes a weak field for the P levels. One of us[6] has extended the application to the $^2P^2D$ multiplet of Cu and the $^3S^3P$ of Be and of Mg. The purpose of this paper is to extend further these calculations to the $^3P^3D$ multiplets as observed in the Zn and Cd spectra and compare the calculated frequencies and intensities with those observed experimentally. Homologous multiplets in these two spectra were chosen to obviate the necessity of changing the field strength and still be able to maintain resolved patterns.

The experimental investigations were made on the 21-foot concave grating set-up at Ohio State University. This set-up employs a 30,000 line concave grating in a Paschen-Runge type mounting. The magnet is of the Weiss type. With ferrocobalt pole tips 1 cm in diameter and a pole gap of about 3 mm, a field strength of 40,000 gauss is obtainable. The light source consists of a spark discharge which takes place in vacuum between a brass wire and the knurled periphery of a revolving disk along which the wire rubbed.[7] A plain brass disk was used to obtain the Zn spectrum and a similar disk having a rim of cadmium for the Cd spectrum. The field strength used was found by calculation from the observed separations of the Zn lines λ4722 and λ4810 whose patterns are accurately known.

Darwin[2] has developed the formula for the positions of the energy levels in a magnetic field of strength $\omega = eHh/4\pi m_0 C$:

$$-a_{m_{l-1},\,m_{s+1}}\beta(l-m_l+1)(s+m_s+1)$$
$$+a_{m_l m_s}[W-\beta\cdot 2m_l m_s-\omega(m_l+2m_s)]$$
$$-a_{m_{l+1},\,m_{s-1}}\beta(l+m_l+1)(s-m_s+1)=0.$$

Where m_l and m_s have their usual significance, β is a constant of the field free multiplet separation, ω is the field strength in cm^{-1} units and W gives the separation in cm^{-1} units of the component from the center of gravity of the multiplet. The a's are numerical coefficients related to the intensities.

The procedure is to write the above equation for all permissible combinations of m_l and m_s to give a certain value of m where $m = m_l + m_s$. Each such set of equations forms a determinant whose solutions are the values of W for that particular value of m. The intensity formulae may be found in Darwin's work. The polarizing action of the grating makes it impossible to compare intensities of parallel components with perpendicular ones.

[1] Heisenberg and Jordan, Zeits. f. Physik **37**, 263 (1926).
[2] Darwin, Proc. Roy. Soc. **A115**, 1 (1927); also K. Darwin, Proc. Roy. Soc. **A118**, 264 (1928).
[3] Kronig, Zeits. f. Physik **31**, 885 (1925).
[4] Fowler, Phil. Mag. **1**, 1079 (1925).
[5] Mensing, Zeits. f. Physik **39**, 24 (1926).
[6] Green, Phys. Rev. **36**, 157 (1930).
[7] See description of vacuum source by Green and Loring in Phys. Rev. **43**, 459 (1933).

TABLE I. *Zinc.*

Position of undisturbed line	Observed position in field	Calculated new theory (1)	Calculated old theory (2)	Obs. int.	Calc. int. (1)	Calc. int. (2)
(P_0-D_1)	−1.50	−1.58	−0.92	10	6.6	13.0
3282.326	−0.55 *p*	−0.59 *p*	0.00 *p*	22	23.2	26.0
30,457.45	0.58	0.57	0.92	12	12.6	13.0
(P_0-D_2)	−1.85	−1.80	fbdn	2	2.8	0
forbidden	0.28 *p*	0.26 *p*	fbdn	2	3.6	0
	2.15	2.21	fbdn	1	0.9	0
(P_0-D_3)	—	−2.60	fbdn	—	0.2	0
forbidden	—	−0.13 *p*	fbdn	—	0.04	0
	—	2.28	fbdn	—	0.02	0
	−3.28	−3.35	−2.76	4	2.2	5.0
	—	−1.41	−0.92	—	0.1	5.0
(P_1-D_1)	−2.11 *p*	−2.20 *p*	−1.84 *p*	4	3.8	10.0
3302.948	−0.54 *p*	−0.60 *p*	fbdn	4	4.0	0
30,267.30	1.28 *p*	1.19 *p*	1.84 *p*	22	22.0	10.0
	0.59	0.56	0.92	6	8.2	5.0
	2.22	2.17	2.76	6	7.4	5.0
	−2.45	−2.47	−2.76	4	5.6	3.0
	−1.73	−1.78	−2.14	25	12.8	9.0
		−1.70	−1.54		17.4	18.0
(P_1-D_2)	−0.49 *p*	−0.52 *p*	−0.62 *p*	20	24.2	18.0
3302.589	0.30 *p*	0.29 *p*	0.00 *p*	14	19.0	24.0
30,270.59	1.01 *p*	0.99 *p*	0.62 *p*	6	4.8	18.0
	1.41	1.39	1.54	20	17.8	18.0
	2.24	2.23	2.14	6	5.0	9.0
	—	3.05	2.76	—	0.6	3.0
	—	−2.52	fbdn	—	0.2	0
	—	−2.23	fbdn	—	0.02	0
	—	−2.00	fbdn	—	0.6	0
	—	−0.05 *p*	fbdn	—	0.8	0
(P_1-D_3)	—	0.29 *p*	fbdn	—	1.2	0
forbidden	—	0.59 *p*	fbdn	—	0.8	0
	2.27	2.35	fbdn	4	4.2	0
	—	2.75	fbdn	—	0.4	0
	—	3.12	fbdn	—	0.2	0

Position of undisturbed line	Observed position in field	Calculated new theory (1)	Calculated old theory (2)	Obs. int.	Calc. int. (1)	Calc int. (2)
	—	−4.95	−4.60	—	0.02	0.4
	—	−3.35	−2.76	—	0.02	0.2
	—	−1.58	−0.92	—	0.2	0.1
(P_2-D_1)	—	−2.20 *p*	−1.84 *p*	—	0.6	0.4
3345.940	—	−0.59 *p*	0.00 *p*	—	0.2	0.6
29,878.41	—	1.19 *p*	1.84 *p*	—	0.2	0.4
	—	0.57	0.92	—	0.6	0.1
	2.22	2.17	2.76	2	1.4	0.2
	4.04	3.94	4.60	2	1.6	0.4
	—	−3.29	−3.38	—	1.4	2.0
	−2.50	−2.48	−2.76	2	1.6	3.0
	—	−1.78	−2.14	—	0.8	3.0
	—	−1.71	−1.54	—	0.4	2.0
	−1.37 *p*	−1.38 *p*	−1.22 *p*	6	4.8	8.0
(P_2-D_2)	—	−0.53 *p*	−0.62 *p*	—	0.02	2.0
3345.572	0.26 *p*	0.28 *p*	fbdn	4	2.2	0
29,881.70	0.99 *p*	0.98 *p*	0.62 *p*	24	6.8	2.0
		1.05 *p*	1.22 *p*		12.6	8.0
	1.43	1.38	1.54	6	4.0	2.0
	2.21	2.23	2.14	4	4.8	3.0
	3.09	3.04	2.76	4	3.6	3.0
	3.71	3.74	3.38	2	1.4	2.0
	—	−2.83	−3.06	—	2.4	1.6
	−2.48	−2.48	−2.76	6	6.6	4.8
	−2.17	−2.19	−2.46	10	11.8	9.6
	−1.94	−1.98	−2.14	14	17.8	16.0
	−1.84	−1.84	−1.84	20	24.0	24.0
	−0.47 *p*	−0.48 *p*	−0.62 *p*	20	19.2	16.0
(P_2-D_3)	−0.07 *p*	−0.07 *p*	−0.30 *p*	24	27.2	25.6
3345.023	0.28 *p*	0.28 *p*	0.00 *p*	20	27.0	28.8
29,886.60	0.58 *p*	0.57 *p*	0.30 *p*	16	20.8	25.6
	0.82 *p*	0.79 *p*	0.62 *p*	8	11.4	16.0
	1.80	1.84	1.84	16	24.0	24.0
	2.30	2.29	2.14	14	14.0	16.0
	2.72	2.69	2.46	6	6.0	9.6
	3.02	3.04	2.76	4	3.0	4.8
	—	3.33	3.06	—	0.8	1.6

For each complete set of data, it was necessary to take four sets of plates. The first was a zinc arc in air together with an iron comparison spectrum, to determine the absolute wave-lengths of the lines of the multiplets. The second set was the complete Zeeman pattern without using any polarizing device. The third set was the perpendicularly polarized components together with the arc in air. This gives the absolute position of the perpendicular components. The fourth set was the parallel polarized components alone. This enabled us to separate out any overlapping of parallel and perpendicular components in the complete pattern of the second set and, through the measurements of the second and third sets, to determine the absolute positions of the parallel components.

$^3P^3D$ MULTIPLET OF ZN

The $^3P^3D$ multiplet of Zn consists of six lines: λλ3345.9, 3345.6, 3345.0, 3302.9, 3302.6, 3282.3. For the ^3P levels $\beta = 579.04/6 = 96.51$ cm^{-1}. For the ^3D levels $\beta = 8.19/10 = 0.819$ cm^{-1}. The field strength $\omega = 1.85$ cm^{-1}. The numerical results are given in Table I. The position of each component is stated in terms of its separation from the field free line in cm^{-1} units. Negative separations indicate frequencies less than that of the parent line. The observed intensities are estimated.

It will be noted that in all but two or three cases the new theory gives much better agreement with the observed positions of the lines than the old. The two or three components for which the positions calculated by the old and new theories fit equally well are cases of weak lines. All cases of unobserved components are lines whose intensities calculate to be very small on the newer theory.

Components which the older theory entirely forbids are especially interesting. The Δj selection rule forbids all components of P_1-D_3, P_0-D_3, and P_0-D_2. One component of P_1-D_3 and all components of P_0-D_2 are observed and check with the Darwin theory. The balance of the P_1-D_3 group and all of the P_0-D_3 lines which are not observed are calculated to have very small

TABLE II. *Cadmium.*

Position of undisturbed line	Observed position in field	Calculated new theory (1)	Calculated old theory (2)	Obs. int.	Calc. int. (1)	Calc. int. (2)
(P_0-D_1)	−1.05	−1.05	−0.91	13	14.0	13.0
3403.653	−0.19 *p*	−0.15 *p*	0.00 *p*	20	21.0	26.0
29,371.76	0.81	0.82	0.91	13	14.0	13.0
(P_0-D_2)	—	−2.05	fbdn	—	0.1	0
forbidden	—	0.06 *p*	fbdn	—	0.1	0
	—	2.16	fbdn	—	0.1	0
(P_0-D_3)	—	−2.36	fbdn	—	0.0	0
forbidden	—	0.09 *p*	fbdn	—	0.0	0
	—	2.50	fbdn	—	0.0	0
	−2.90	−2.88	−2.73	4	4.0	5.0
	−1.10	−1.05	−0.91	4	3.3	5.0
(P_1-D_1)	−1.90 *p*	−1.93 *p*	−1.82 *p*	6	7.1	10.0
3467.656	—	0.15 *p*	fbdn	—	0.0	0
28,829.59	1.70 *p*	1.68 *p*	1.82 *p*	10	13.3	10.0
	0.82	0.80	0.91	8	7.7	5.0
	2.55	2.58	2.73	8	6.0	5.0
	−2.62	−2.67	−2.73	3	4.3	3.0
	−2.05	−2.05	−2.12	10	10.8	9.0
	−1.58	−1.57	−1.52	17	18.0	18.0
(P_1-D_2)	−0.57 *p*	−0.57 *p*	−0.61 *p*	20	21.0	18.0
3466.201	0.09 *p*	0.06 *p*	0.00 *p*	20	23.4	24.0
28,841.70	0.68 *p*	0.68 *p*	0.61 *p*	17	14.7	18.0
	1.46	1.48	1.52	17	18.0	18.0
	2.10	2.16	2.12	7	7.3	9.0
	2.79	2.79	2.73	1	2.0	3.0
	—	−2.64	fbdn	—	0.03	0
	—	−2.36	fbdn	—	0.3	0
	—	−2.07	fbdn	—	0.1	0
	—	−0.23 *p*	fbdn	—	0.1	0
(P_1-D_3)	—	0.09 *p*	fbdn	—	0.1	0
forbidden	—	0.37 *p*	fbdn	—	0.1	0
	—	2.16	fbdn	—	0.1	0
	—	2.50	fbdn	—	0.03	0
	—	2.82	fbdn	—	0.03	0

Position of undisturbed line	Observed position in field	Calculated new theory (1)	Calculated old theory (2)	Obs. int.	Calc. int. (1)	Calc. int. (2)
	—	−4.67	−4.55	—	0.3	0.4
	—	−2.88	−2.73	—	0.03	0.2
	—	−1.05	−0.91	—	0.0	0.1
(P_2-D_1)	—	−1.93 *p*	−1.82 *p*	—	0.7	0.4
3614.4	—	−0.15 *p*	0.00 *p*	—	0.6	0.6
27,658.96	—	1.68 *p*	1.82 *p*	—	0.3	0.4
	—	0.80	0.91	—	0.3	0.1
	—	2.58	2.73	—	0.6	0.2
	—	4.42	4.55	—	0.8	0.4
	−3.32	−3.30	−3.34	2	2.0	2.0
	−2.64	−2.67	−2.73	3	2.6	3.0
	−2.04	−2.05	−2.12	3	2.6	3.0
	−1.57	−1.57	−1.52	2	1.3	2.0
	−1.23 *p*	−1.25 *p*	−1.21 *p*	5	6.5	8.0
(P_2-D_2)	−0.55 *p*	−0.57 *p*	−0.61 *p*	1	1.3	2.0
3612.875	—	0.06 *p*	fbdn	—	0.0	0.0
27,670.63	0.68 *p*	0.68 *p*	0.61 *p*	2	3.3	2.0
	1.16 *p*	1.16 *p*	1.21 *p*	8	9.3	8.0
	1.42	1.48	1.52	2	2.7	2.0
	2.09	2.16	2.12	5	3.3	3.0
	2.76	2.79	2.73	5	3.3	3.0
	3.38	3.41	3.34	3	2.0	2.0
	−2.96	−2.96	−3.13	2	2.0	1.6
	−2.63	−2.64	−2.73	5	4.7	4.8
	−2.34	−2.35	−2.43	8	10.0	9.6
	−2.08	−2.07	−2.12	14	16.7	16.0
	−1.80	−1.82	−1.82	20	24.0	24.0
	−0.56 *p*	−0.57 *p*	−0.62 *p*	14	17.3	16.0
(P_2-D_3)	−0.21 *p*	−0.23 *p*	−0.31 *p*	24	26.7	25.6
3610.510	0.09 *p*	0.09 *p*	0.00 *p*	24	28.6	28.8
27,688.75	0.38 *p*	0.37 *p*	0.31 *p*	20	24.6	25.6
	0.69 *p*	0.66 *p*	0.62 *p*	16	14.7	16.0
	1.84	1.82	1.82	24	24.0	24.0
	2.14	2.16	2.12	16	15.3	16.0
	2.46	2.49	2.43	9	8.7	9.6
	2.80	2.82	2.73	5	4.0	4.8
	3.07	3.10	3.13	1	1.3	1.6

intensities. The older theory also forbids any transition for which $m=0\rightarrow m=0$ when Δj equal zero. Examples of this are seen in the center parallel components of P_2-D_2 and P_1-D_1. In both cases lines are observed and show good agreement. The components at 0.98 and 1.05 in the P_2-D_2 group are unresolved and the line observed at 0.99 shows an intensity comparable with the sum of the two. A similar case occurs in the line observed at -1.73 in the P_1-D_2 group.

$^3P^3D$ Multiplet of Cd

Results of similar investigation of the same multiplet in Cd are shown in Table II and Figs. 1b and 2b. The six lines of this multiplet occur at λλ3614.4, 3612.9, 3610.5, 3467.7, 3466.2, 3403.7. For the ^3P levels $\beta=1712.8/6=285.47$ cm^{-1} and for the ^3D levels equals $29.8/10=2.98$ cm^{-1}. The field strength $\omega=1.82$ cm^{-1}.

There will be noted in these results a much closer agreement between old and new theory than in the previous case. This is to be expected since the constants of multiplet separation here are much larger, some 3.5 times as large for the ^3D levels and 3 times as large for the ^3P. In this case, then, the field of approximately the same strength as before comes nearer being a weak field, i.e., one in which the multiplet separation is much greater than the normal Zeeman separation.

However, here also in most cases the new theory shows better agreement with observation. For seven lines the two theories give about equal agreement and for two—the 2.10 component of P_1-D_2 and the 2.09 component of P_2-D_2—the older theory checks slightly better with experiment.

No check can be made here on forbidden lines since all lines forbidden by old theory have a small calculated intensity by new theory and none were observed in these comparatively short exposures.

Conclusions

The intervals of the ^3D levels of both Zn and Cd are extremely close to 3 : 2 and the ^1D levels are sufficiently far away so that we may assume that we have cases of Russell-Saunders (*L-S*) coupling. The experimental evidence of the present paper

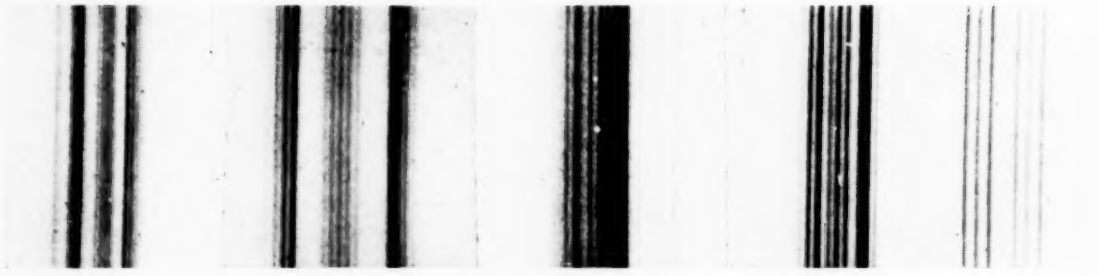

Fig. 1a Fig. 1b Fig. 2a Fig. 2b

FIG. 1a. Zn 3345 $^3P_2-{}^3D_2$ (second order)
FIG. 1b. Cd 3610 $^3P_2-{}^3D_2$ (third order)
FIG. 2a. Zn 3302–3 $^3P_1-{}^3D_2$ and $^3P_1-{}^3D_1$ (second order)
FIG. 2b. Cd 3466–8 $^3P_1-{}^3D_2$ and $^3P_1-{}^3D_1$ (third order)

The figures show clearly that the first evidence of the beginning Paschen-Back effect is a distortion of intensities, as shown in the case of the cadmium lines, while with greatly increased field strengths (relative to the multiplet separations) the positions of the components are so badly distorted as almost to destroy the symmetry of the pattern.

shows that the Darwin theory is quite adequate for the calculation of the positions and intensities of the Zeeman components. The first noticeable effect of the increasing magnetic field is a distortion of intensities. This is the case with Cd. It is not until the field becomes quite strong that the field distorts the positions of the lines very markedly. (See Figs. 1 and 2.)

In the spectrum of mercury, *L-S* coupling is practically completely destroyed and the Darwin theory is no longer adequate for the explanation of the experimental evidence. The results for mercury will be published shortly.

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The Vibrational Energy Level System of the Linear Molecule HCN

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The discovery of new rotation-vibration bands in the infrared absorption spectrum of gaseous HCN has made possible a determination of the vibrational energy level system of the normal molecule. The agreement between theory and experiment is completely satisfactory. The fundamental frequencies of the isotopic molecule H^2CN are predicted from those found for H^1CN.

INTRODUCTION

ONE of the most important problems entailed in the analysis of the infrared spectrum of a molecule is the accurate determination of the vibrational energy level scheme from the observed absorption centers. In addition to providing the correlating network which serves to classify new absorption regions of the molecule unambiguously, the energy diagram contributes information which is of importance for other problems in physics and chemistry. Thus, the direct calculation of the thermodynamic potentials demands a precise knowledge of the vibrational states of the molecule.[1] Also, the coefficients in the expression for the position of an oscillational level serve to describe the potential function which governs the anharmonic vibrations of the atoms.[2] In this report it is proposed to set forth the energy level diagram of the linear molecule H−C−N.

ANALYSIS

In proceeding with the analysis, we shall employ the expression:

$$E_{\text{vib}}=\nu_1 V_1+\nu_2 V_2+\nu_3 V_3+x_{11}V_1^2+x_{22}V_2^2 +x_{33}V_3^2+x_{ll}l^2+x_{12}V_1V_2+x_{13}V_1V_3 +x_{23}V_2V_3,$$

which gives the energy of vibration of the linear triatomic molecule correct through a second order perturbation calculation on the anharmonic potential function. Here, V_1, V_2, V_3 and l are the quantum numbers of vibration, and $\nu_1\cdots x_{23}$ are functions of the harmonic and anharmonic constants of the potential energy of the normal molecule.[2] Quite clearly, the energy diagram may be constructed as soon as the quantities $\nu_1\cdots x_{23}$ become available. These, in turn, may be found from the set of ten simultaneous equations founded upon the application of the expression for E_{vib} to the positions of ten vibration levels as deduced from infrared and Raman measurements.

APPLICATION TO HCN

The solution of this problem for the HCN molecule was made possible by the experimental work discussed in the next paragraph, specifically by the determination of the combination bands $\nu_2+\nu_3$ and $2\nu_1+\nu_2$.

A metal tube 150 cm long closed with thin glass windows served as the absorption cell. After being evacuated it was filled with saturated vapor of HCN and introduced into the optical

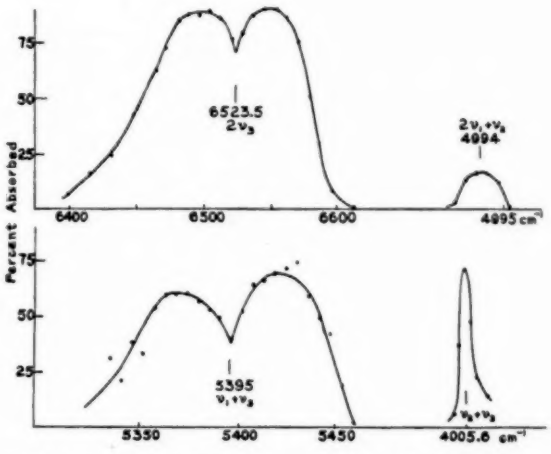

FIG. 1. Near infrared absorption bands of HCN.

[1] L. Kassel, Phys. Rev. **43**, 364 (1933).

[2] Adel and Dennison, Phys. Rev. **43**, 716; **44**, 99 (1933).

path of a prism grating spectrometer, the grating having 4800 lines per inch. This gives sufficient resolution to show the band contours clearly, but does not resolve the rotational structure. Fig. 1 shows the four weak bands which have been

TABLE I. *Observed and calculated positions of some infrared lines of HCN.*

V_1	V_2	V_3	l	Position Calculated	Observed	Observer
0	0	0	0	0.0 cm^{-1}	0.0 cm^{-1}	
0	1	0	1	712.3	712.3 *	Choi and Barker[3]
0	2	0	0	1412.6	1412.4 *	Choi and Barker[3]
0	2	0	2	1425.0		
1	0	0	0	2089.0	2089.0 *	Kastler[4]
0	3	0	1	2113.3	2117.3	Barker[5]
1	1	0	1	2801.1	2801.1 *	Choi and Barker[3]
0	4	0	0	2802.8	2800.0	Barker[5]
0	4	0	2	2815.2	2815.0 *	Barker[5]
0	0	1	0	3312.9	3312.9 *	Adel and Barker
1	2	0	0	3501.2		
1	2	0	2	3513.6		
0	1	1	1	4005.6	4005.6*	Adel and Barker
1	3	0	1	4201.7		
2	0	0	0	4282.0		
0	2	1	0	4686.3		
0	2	1	2	4698.7		
2	1	0	1	4993.9	4993.9 *	Adel and Barker
0	3	1	1	5367.4		
1	0	1	0	5386.8	5395 (?)†	Adel and Barker
2	2	0	0	5693.8		
2	2	0	2	5706.2		
0	0	2	0	6523.2	6523.5	Adel and Barker
3	0	0	0	6579.0		
1	2	1	0	6759.8		
1	2	1	2	6772.2		
0	1	2	1	7196.3		
3	1	0	1	7290.7		
2	0	1	0	7564.7		
0	2	2	0	7857.4		
0	2	2	2	7869.8		
2	1	1	1	8257.0		
1	0	2	0	8582.0	8591 (?)	Brackett and Liddel[6]
4	0	0	0	8980.0		
1	1	2	1	9254.9		
0	0	3	0	9630.9	9645 (?)	Brackett and Liddel[6]
3	0	1	0	9846.6		
1	2	2	2	9928.2		
0	1	3	1	10284.4		
2	0	2	0	10744.8		
0	2	3	2	10938.3		
1	0	3	0	11674.5	11674.5 *	Badger and Binder[7]
1	1	3	1	12327.9		
0	0	4	0	12635.8	12635.8 *	Badger and Binder[7]

† The intense water vapor background absorption in this region rendered precise measurements on $\nu_1+\nu_3$ very difficult.

[3] Choi and Barker, Phys. Rev. **43**, 777 (1932).
[4] C. R. Kastler, Comptes Rendus **194**, 858 (1932).
[5] E. F. Barker, Phys. Rev. **23**, 200 (1924).
[6] Brackett and Liddel, Smithsonian Institution **85**, No. 5 (1931).
[7] Badger and Binder, Phys. Rev. **37**, 800 (1931).

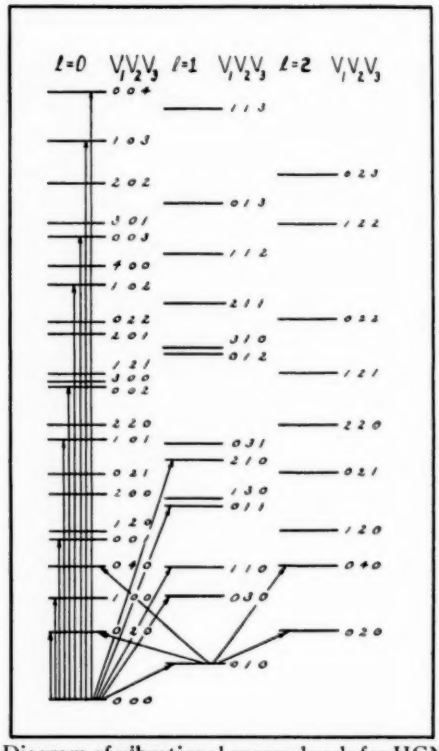

FIG. 2. Diagram of vibrational energy levels for HCN showing transitions which have been observed.

identified. (It seemed desirable at this time to redetermine the band center for ν_3 at 3μ as there is an obvious error in the position indicated in reference 3. The midpoint of the reported maxima lies at 3311 cm^{-1} while the band center is listed at 3289 cm^{-1}. For this measurement a short cell with salt windows was used and just enough gas to give rather weak absorption maxima. The central absorption minimum was found to lie at 3312.9 cm^{-1}.)

In Table I, column III lists the observed infrared and Raman spectra of gaseous HCN. The ten levels marked with asterisks are those used in the solution for the energy constants $\nu_1 \cdots x_{23}$ the resulting numerical values being

$\nu_1 = 2037.0\ \text{cm}^{-1}$, $x_{11} = 52.0\ \text{cm}^{-1}$, $x_{12} = -0.2\ \text{cm}^{-1}$,
$\nu_2 = 712.1$, $x_{22} = -2.9$, $x_{13} = -15.1$,
$\nu_3 = 3364.2$, $x_{33} = -51.3$, $x_{23} = -19.6$.
$x_{ll} = 3.1$,

Column II gives some of the more important vibrational levels as computed from these energy constants. From the comparison between observed and computed positions, it would seem of interest to remeasure the band $3\nu_3$.

The computed energy diagram, drawn to scale, and the eighteen transitions which have already been observed are indicated in Fig. 2.

Conclusion

We have now completely determined the vibrational energy scheme of the molecule of gaseous HCN, thus making it possible to classify at once new infrared and Raman data. The application of these results to a discussion of the specific heat, free energy and other thermodynamic quantities characteristic of HCN is in progress, and the results should appear in the near future.

Note on the Isotope Molecule H^2CN

In conformity with the assumption of valence forces acting within the molecule and with the second assumption that these forces are invariant for an isotopic change in mass, the data ν_1, ν_2 and ν_3 of normal HCN suffice for the calculation of the fundamental frequencies of isotopic H^2CN. In this manner we find for H^2CN: $\nu_1 \cong 1920$ cm^{-1}; $\nu_2 \cong 570$ cm^{-1}; $\nu_3 \cong 2580$ cm^{-1}, probably correct to within five percent.

It is of interest to note that whereas it has been impossible to observe ν_1 in the infrared spectrum of HCN because it lies buried beneath the mass of absorption $3\nu_2$ and $4\nu_2 - \nu_2$, the above calculation indicates that ν_1 and $3\nu_2$ of H^2CN may be expected to lie about 100 cm^{-1} apart. An attempt to locate the fundamentals of H^2CN is projected in order to obtain data which will contribute to a description of the potential function of the molecule.

LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the twentieth of the preceding month; for the second issue, the fifth of the month. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

The Absorption Spectrum of Diatomic Antimony (Sb_2)

The fluorescence spectrum of Sb_2 has recently been studied by Genard.[1] As Genard points out the absorption spectrum of the Sb_2 molecule has not been investigated before.

0.2 gram of pure antimony metal was sealed off in an evacuated quartz tube provided with plane-sealed-in quartz windows. This tube which served as an absorption vessel was 40 cm long and 2.5 cm in diameter. It also had a side tube through which the absorption tube could be evacuated and this tube was sealed off about 20 cm from the main tube. The absorption tube was inserted into an electric oven which could be heated to 1050°C. A second electric oven was built round the side tube. Both ovens were provided with calibrated thermocouples. The temperatures of both ovens could be kept at any desired temperature by regulating the currents flowing through them by means of resistances.

A hydrogen tube was used as a continuous source, the power being supplied by a 5 kw transformer.

Test plates were first taken with a Hilger E3 spectrograph with the temperature of the main tube varying between 600 and 1050°C while the temperature of the side tube was kept fixed at 600, 700 and 800°C. In this way the pressure of the vapor and the number of antimony molecules in the light path could be varied. It was found that with the side tube at 600°C and the main tube at 800°C an intense absorption spectrum appeared between λ2300 and λ2200. This spectrum could not be extended by raising the temperature of either tube, but as the pressure of the vapor was increased, complete absorption set in and spread to longer wave-lengths with increasing pressure.

A more extended band system was observed between 3300 and 2840 when the temperature of the main tube was increased to 1000°C, that of the side tube being 700 or 800°C. This spectrum was very sensitive to changes of temperature or pressure of the antimony vapor in the main tube. This band system has been photographed with a Hilger E1 spectrograph and analyzed. The provisional equation representing the heads of the bands observed may be given as:

$$=31{,}605+[212(v'+\tfrac{1}{2})-0.2(v'+\tfrac{1}{2})^2]$$
$$-[268(v''+\tfrac{1}{2})-0.6(v''+\tfrac{1}{2})^2].$$

The constants of the molecule corresponding to the lower state of this system agree with those given by Genard. The upper state appears to be perturbed in the same way as Christy and Naudé[2] first observed in the spectrum of the S_2 molecule. The band head at λ2842 is diffuse and for shorter wave-lengths the spectrum becomes very diffuse. This must probably be ascribed to predissociation.

According to Aston[3] antimony has two isotopes: Sb^{121} and Sb^{123} which have the abundance ratio 10 : 8 approximately. One would therefore expect to observe bands corresponding to the molecules $(Sb^{121})_2$, $Sb^{121}\cdot Sb^{123}$, and $(Sb^{123})_2$. If one takes account of the fact that the first and third types of molecule will only have half the number of rotational lines the second type of molecule has, the intensity ratio of the band heads corresponding to the three types of molecule should be 50 : 80 : 30, respectively. Actually the bands at λλ2858, 2877, 2886, 2895 and 2906 have been observed to show a distinct fine structure which agrees within experimental error with the calculated fine structure. Moreover the head corresponding to the $Sb^{121}\cdot Sb^{123}$ molecule is the most intense of the three in every case.

A preliminary analysis of the first-mentioned band system indicates that it probably has its lower state in common with the upper state of the latter system.

The work is being continued with a view to establish the perturbations in the upper state with greater accuracy.

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December 28, 1933.

[1] J. Genard, Phys. Rev. **44**, 468 (1933).
[2] A. Christy and S. M. Naudé, Phys. Rev. **37**, 903 (1931).
[3] F. W. Aston, Proc. Roy. Soc. **A132**, 487 (1931).

Flame and Spark-in-Flame Spectra of Rare Earths

While attempting to use the spark-in-flame method of Hultgren[1] for the analysis of rare earth salts, spectrograms of the flame were taken with rare earth salts introduced in the form of a spray. It was found that lanthanum,

[1] Hultgren, J. A. C. S. **54**, 2324 (1932).

samarium, praseodymium, neodymium, gadolinium, ytterbium gave a series of bands.

Spectrograms were also taken with a spark passing through the flame directly over the inner cone. A Thordarson 1 kilowatt 20,000-volt transformer was used. The spark gap electrodes were made of 1 mm tungsten wire. The condenser was made with two 600 cc beakers used as Leyden jars. The auxiliary spark gap used by Hultgren was omitted. The photographic plates showed the same bands as in the flame alone but the time of exposure was considerably shortened. Solutions of the acetate, nitrate, chloride and sulfates were used all giving the same bands insofar as we were able to ascertain. A Hilger constant deviation spectrograph with a low dispersion prism was used.

With $LaCl_3$, 19 bands were observed with heads at 4372, 4379, 4383, 4418, 4423, 4428, 4433, 4437, 4442, 5378, 5403, 5428, 5600, 5628, 5650, 5675, 5706, 5923, and 5957A. These bands were obtained in the spectrum of LaO reported by Meggers and Wheeler[2] in their paper in which they report some 300 band heads.

With $PrCl_3$ seven bands with heads at 6015, 5950, 5755, 5743, 5715, 5685, 5528, and a sharp line at 4223 were observed.

Neodymium acetate gave 26 bands with heads at 4369, 4377, 5475, 5565, 5625, 5973, 5993, 6010, 6266, 6278, 6295, 6347, 6365, 6382, 6415, 6425, 6441, 6460, 6492, 6513, 6533, 6546, 6580, 6598, 6623, and 6647.

$SmCl_3$ gave bands at 4338, 4444, 5520, 5617, 5808, 5818, 5847, 5868, 5908, 5924, 5947, 5964, 5983, 6015, 6032, 6047, 6066, 6087, 6108, 6138, 6150, 6163, 6182, 6223, 6242, 6253, 6333, 6344, 6388, 6405, 6417, 6455, 6480, 6506, 6528, 6553, 6575, 6615, 6646, 6679, and 6702.

$GdCl_3$ gave 16 bands at 6232, 6214, 6192, 6171, 6147, 6126, 6098, 6077, 6055, 5982, 5952, 5935, 5917, 5676, 5692, 5662.

$YbCl_3$ gave bands which were very fuzzy at 4682, 4768, 4858, 4985, 5183, 5340, 5460, 5542, 5758, 5867, 6540. A line at 5556 and a doublet at 3986–3988 were also observed.

With YCl_3 Megger's and Wheeler's[2] system III transition $^2\Pi_{\frac{1}{2}} - {}^2\Sigma_{\frac{1}{2}}$ and system $V\,{}^2\Pi_{\frac{1}{2}} - {}^2\Pi_{\frac{1}{2}}$ with the heads at 5972 and 6132 respectively were the most prominent. Weaker bands at 4650, 4676, 4706, 4817, 4842, 5694, 5714, 5730, 5747, 5764, and 5783 were also obtained. Piccardi[3] has obtained bands with La_2O_3, Pr_2O_3, Nd_2O_3 and Sm_2O_3 which he attributes to the MO molecule, M being the rare earth metal. Since the spectrograph used had low dispersion, no analysis of the bands was attempted.

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January 10, 1934.

[2] Meggers and Wheeler, Bur. Standards J. Research **6**, 239 (1931).

[3] Piccardi, Nature **124**, 129 (1929).

The Concentration of H^3 and O^{18} in Heavy Water

A large amount of work in recent years on the two well-known hydrogen isotopes has led to much speculation on the existence of a third isotope of mass 3. Lewis and Spedding[1] concluded that ordinary hydrogen contained less than one part of H^3 in six million. Since then Latimer and Young,[2] using the magneto-optic method of Allison, have found evidence for this isotope in 2 percent heavy water. An attempt has been made to confirm this conclusion with the mass-spectrograph without success. The results are presented graphically in Fig. 1. The partial pressure of deuterium as measured by the intensity of the $(H^2H^2)^+$ peak is plotted horizontally and the ordinates represent the intensity of some other peak divided by this partial pressure. This procedure has been described at some length in a previous paper.[3] The upper curve represents this relation for the triatomic ion $(H^2H^2H^2)^+$ of mass 6. Since the experimental points may be accurately fitted to a straight line passing through the origin this curve constitutes a proof that at low pressures the number of triatomic ions formed is proportional to the square of the pressure. In previous work this fact was assumed without proof in this method of analysis. It is significant also that the experimental points for this curve were obtained from three different samples varying in concentration from 70 to 98 percent deuterium.

Curves II and III represent in a similar way the data for ions of mass 5 which may exist both as $(H^2H^3)^+$ and $(H^1H^2H^2)^+$. The intercept on the vertical axis should give the abundance ratio $H^2H^3 : H^2H^2$ which within the limits

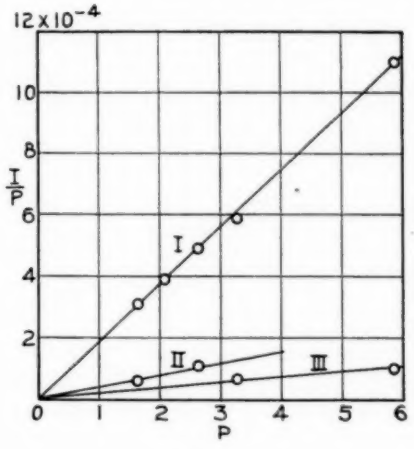

FIG. 1.

[1] G. N. Lewis and F. H. Spedding, Phys. Rev. **43**, 964 (1933).

[2] W. M. Latimer and H. A. Young, Phys. Rev. **44**, 690 (1933).

[3] W. Bleakney, Phys. Rev. **41**, 32 (1932).

of measurement is zero. The hydrogen which yielded the data for curve II was prepared by dissociating heavy water on hot iron and on analysis showed a concentration of 91 percent deuterium. The sample for curve III was obtained by electrolysis of nearly pure heavy water and in this case a concentration of 98 percent was found. These two curves do not coincide because as we go to samples richer in deuterium there is less chance of the formation of triatomic ions of mass 5. From these results we conclude that $H^3 : H^2$ is less than one in 10^5 in the samples tested. This means that in ordinary hydrogen $H^3 : H^1$ is less than one in 5×10^8 if we take our previous value[4] for the $H^2 : H^1$ ratio of 1 : 5000 and assume there has been no fractionation at all between H^3 and H^2 upon electrolysis of water. Actually, we would expect some separation so that H^3 must be extremely rare in natural hydrogen. Since the great sensitivity claimed for the magneto-optic method reaches considerably beyond this limit there is no essential contradiction between the conclusions of Latimer and Young and those presented here.

From the same cell which yielded 98 percent electrolytic deuterium a sample of oxygen was collected and examined for increased concentration of the O^{18} isotope. A control run was carried out at the same time with electrolytic oxygen from ordinary water. No increase in the $O^{18} : O^{16}$ ratio could be observed although a ten percent change should have been detectable. We are unable to reconcile this result with the work of Washburn, Smith and Frandsen.[5]

WALKER BLEAKNEY
AUSTIN J. GOULD

Palmer Physical Laboratory and
Frick Chemical Laboratory,
Princeton, New Jersey,
January 30, 1934.

[4] W. Bleakney and A. J. Gould, Phys. Rev. **44**, 265 (1933).
[5] E. W. Washburn, E. R. Smith and M. Frandsen, Bur. Standards J. Research **11**, 453 (1933).

About the Production of "Colloidal Single Crystals"

Professor K. S. Krishnan[1] has recently pointed out that the method employed by us[2, 3] to fix small suspended particles of anisotropic substances in a position of mutual crystallographic parallelity fails in case of graphite. This objection is perfectly justified by the fact that we have not yet published the method in detail. In order to avoid similar doubts it may thus be mentioned in which way it is possible to line up small particles of a crystalline substance which has more than *one* axis of maximum paramagnetism, like graphite (in contradistinction to, for instance Bi, where there is more than one direction of maximum diamagnetism but only one of maximum paramagnetism [111]). Particles of the former type will line themselves up in a liquid suspending medium when exposed to a magnetic field *only with reference to one direction*—one of the directions of maximum paramagnetism—, i.e., for graphite the (0001) planes will be parallel to the lines of force, though the axis [0001] can be orientated in any azimuthal direction. This azimuthal orientation can be

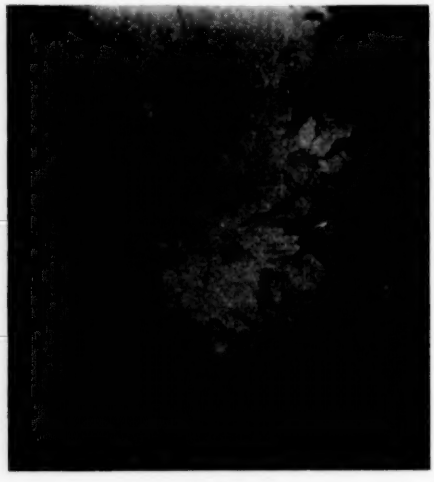

FIG. 1.

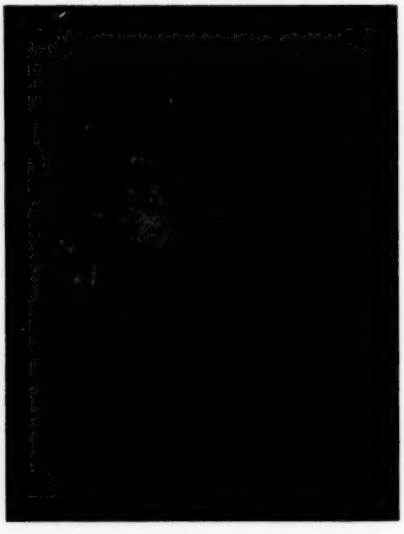

FIG. 2.

[1] K. S. Krishnan, Phys. Rev. **45**, 115 (1934).
[2] A. Goetz, A. B. Focke and A. Faessler, Phys. Rev. **39**, 169, 553 (1932).
[3] A. Goetz and A. Faessler, Phys. Rev. **40**, 1053A (1932).

unified, however, by *rotating the container of the suspension within the magnetic field around an axis parallel to the direction of the [0001] axis which one wishes to obtain for the suspension.*[4] It is obvious that by a turn around 90° the particles whose (0001) planes have not been parallel to the field will be acted upon by a ponderomotoric force, correcting for this deviation. This effect is amplified greatly by bringing the particles gradually into an inhomogeneous region of the magnet. It is thus possible to line up all particles in both directions in a few minutes. Only then the suspending medium is hardened. The microphotographs Figs. 1 and 2 show such hardened gelatine suspensions of large graphite flakes, obtained without and with rotation and seen in the direction of the lines of force.

Referring to the remark of Professor Krishnan about the "more or less uniform" size of the particles, it may be mentioned tnat greatest care was taken by fractionated sedimentation and centrifugation to have the variation of sizes of the particles not larger than 5 percent.[5]

ALEXANDER GOETZ

California Institute of Technology,
January 31, 1934.

[4] This idea originated with Dr. A. Faessler first for which contribution I wish to give him full credit.

[5] A detailed description of the methods and results will soon be published in the Zeitschrift für Physik.

Errata.—The Production of Positives by Nuclear Gamma-Rays

(PHYS. REV. **44**, 948, 1933)

The factor preceding the integral in Eq. (2) should read $\alpha/3\pi\gamma^5$ instead of $\alpha/\pi\gamma^5$.

In the second column in the tenth line of the first paragraph the expected yield of positives should be about 6×10^{-8} instead of 6×10^{-9} per alpha-particle.

LEO NEDELSKY
J. R. OPPENHEIMER

University of California.

Proceedings of the American Physical Society

MINUTES OF THE BOSTON MEETING, DECEMBER 28–30, 1933

THE 35th Annual Meeting (the 189th regular meeting) of the American Physical Society was held at Boston on Thursday, Friday and Saturday, December 28, 29 and 30, 1933, in affiliation with Section B—Physics—of the American Association for the Advancement of Science. The presiding officers were Dr. Paul D. Foote, President of the Society, Professor A. H. Compton, Vice-President, Dr. L. J. Briggs, Dr. K. K. Darrow, Dr. Thomas H. Johnson, Professor F. W. Loomis and Professor G. W. Pierce. There were more than five hundred physicists in attendance. Sessions were held at both Harvard University and the Massachusetts Institute of Technology.

On Thursday afternoon there was a symposium on *Nuclear Physics*. The speakers at this symposium were (1) Professor G. E. Uhlenbeck of the University of Michigan on *Introduction to the Theory of the Positron;* (2) Dr. Carl D. Anderson of the California Institute of Technology on *The Positron;* and (3) Professor J. R. Oppenheimer of the University of California on *The Theory of the Electron and Positive.* This proved to be a session of great interest and importance, and the attendance was about five hundred.

The annual joint session with Section B was held on Friday morning. The presiding officer was Dr. C. J. Davisson, Vice-President of Section B. There was an address by Dr. Paul D. Foote, President of the American Physical Society, who delivered his presidential address on *Industrial Physics.* The Retiring Vice-President of Section B, Professor D. L. Webster, delivered an address on *Current Progress in X-Ray Physics.* An address was also given by Professor James Franck formerly of the University of Göttingen on *Hydrogen in Palladium.*

Annual Business Meeting: The regular annual business meeting of the American Physical Society was held on Friday morning, December 29, 1933, in the New Lecture Hall of Harvard University. The meeting was presided over by President Foote. The President had appointed Messrs. G. Breit and I. I. Rabi to canvas the ballots for officers of the Society. They reported the following elections for the year 1934:

President	Arthur H. Compton
Vice-President	R. W. Wood
Secretary	W. L. Severinghaus
Treasurer	George B. Pegram
Members of the Council—four year term	E. O. Lawrence F. K. Richtmyer
Members of the Board of the Physical Review—three year term	Alexander Ellett Louis A. Turner George E. Uhlenbeck

The Secretary reported that during the year there had been 133 elections to membership. The deaths of 10 members had been reported during the year; 38 had resigned; and 4 had been dropped. The membership of the Society as of December 27, 1933 was as follows: Members: 2005; Fellows: 662; Honorary Members: 6; Total Membership: 2673.

The Treasurer presented a summary of the financial condition of the Society. It was impossible to present a final report for the year at the Annual Business Meeting because the fiscal year ends on December 31st. The Treasurer's financial report will be audited, printed and distributed.

The Managing Editor reported that it was no longer his duty to report the financial status of the various publications of the Society since this is now being done by the American Institute of Physics. He reported that during the first half of the past year the average length of papers was 6.93 pages and during the second half 6.19 pages. The advantages and disadvantages of attempting to shorten the papers were summarized. Among

other things it resulted in an acculumation of unprinted papers in the hands of the authors for correction.

On Friday noon Harvard University very kindly invited the members of the Physical Society to be its guests at luncheon in the Harvard Union.

The annual dinner of the Society was held on Friday evening at seven o'clock at the Parker House. There were 141 present. Dr. Paul D. Foote presided and introduced the new president, Professor Arthur H. Compton. The after-dinner speakers were Dr. K. K. Darrow, Professor Duane Roller, Editor of the American Physics Teacher, and Dr. W. F. G. Swann.

Meeting of the Council: At the meeting of the Council held on Thursday, December 28, 1933, one candidate was elected to fellowship, three candidates were transferred from membership to fellowship, and twenty were elected to membership. *Elected to Fellowship:* Otto Stern. *Transferred from Membership to Fellowship:* Walter H. Brattain, Morris Muskat and Brian O'Brien. *Elected to Membership:* J. M. Benson, Jonathan Biscoe, T. W. Bonner, Carl A. Cinnamon, Byron E. Cohn, Paul M. Gross, Elmo E. Hanson, Clement L. Henshaw, Lawrence E. Kinsler, Esther U. Mintz, K. H. Moore, Daniel Norman, D. A. Quarles, William H. Ross, E. H. Schrieber, William L. Smith, S. Town Stephenson, D. A. Wilbur, Karl S. Woodcock, and W. W. Woodcock, Jr.

The regular scientific program of the Society consisted of sixty-four papers. Numbers 1, 3, 56, 57, 58, and 59 were read by title. The abstracts of these papers are given in the following pages. An *Author Index* will be found at the end.

W. L. SEVERINGHAUS, *Secretary*

ABSTRACTS

1. The Polarization of Light at Sea. E. O. HULBURT, *Naval Research Laboratory.*—Measurements have been made of the polarization of the light of the surface of the sea for a variety of weather conditions. From the experimental results and theory explanations emerge of the character of the sea reflection, the width of the sun path on the water and a number of familiar phenomena. A polarizing prism properly oriented darkens the sea relative to the sky, reduces the brilliancy of the sun path and renders the horizon more distinct. Attaching polarizing prisms to a sextant and to binoculars improved these instruments in certain cases.

2. The Rapid Derivation of Thermodynamical Relations for a Simple System. A. NORMAN SHAW, *McGill University.*—A procedure has been developed whereby relations involving first and second partial derivatives may be deduced with great rapidity. It has been possible to construct three compact tables which lead to the relations between derivatives chosen in any manner whatever. Previously available tables have been confined mainly to transformations involving only a few sets of those independent derivatives most commonly chosen for reference. The methods are based on the use of a Jacobian notation which leads to much simplification in transforming partial derivatives. The following equations are shown to be adequate for deriving rapidly the major portion of the myriad of possible thermodynamical relations for a simple system,—

$$J(x, y)\cdot J(z, w)+J(y, z)\cdot J(x, w)+J(z, x)\cdot J(y, w)=0$$

$$J[J(x, y), z]+J[J(y, z), x]+J[J(z, x), y]=0,$$

where x, y, z, w may be any thermodynamical variables considered under circumstances where any three of them determine the physical state of a system. These equations are combined with $J(p, v)=J(T, \phi)$; and in applications, the following is particularly useful,—

$$(\partial x/\partial y)_z=\partial(x, z)/\partial(y, z)=J(x, z)/J(y, z)$$
$$=(A_xB_z-A_zB_x)/(A_yB_z-A_zB_y),$$

where $A_r=(\partial r/\partial\alpha)_\beta$, and $B_r=(\partial r/\partial\beta)_\alpha$, in which α and β represent any two permissible independent variables, and r is any dependent variable.

3. A Note on the Ratio of the Thermal Coefficient of Expansion to the Specific Heat at Constant Pressure for Tungsten. WILFRID J. JACKSON, *Rutgers University, New Brunswick, New Jersey.*—The variation of the ratio α/C_p with temperature has been worked out for tungsten by using the recent careful measurements and the empirical relationship obtained from the data of Bronson, Chisholm and Dockerty. (Canadian Journal of Research **8**, 282–303 (1933)). The best available data on the thermal coefficient of expansion are used. It is found that the ratio is a constant for only a limited range of temperature for which the empirical relationships are applicable. Above 200°C the ratio increases continuously with temperature up to 500°C where the validity of the relationships ceases. A table is given showing the variation in the ratio.

4. The Science of Sealing Metals to Glass. ALBERT W. HULL, *General Electric Company, Schenectady, N. Y.*—The differential thermal expansion between glass and metal has

been measured with great care for all the common sealing materials and for a number of new alloys and glasses. Typical combinations have then been tested, after annealing at definite temperatures, for strength and strain, and the results compared with predictions based on the expansion measurements. A new alloy is described, whose expansion matches that of a particular glass over the entire range from zero to the softening point. With this combination of glass and metal it is possible to test the hypothesis, which is basic to a science of seal-making, that properly annealed seals will be strain-free to exactly the degree indicated by the expansion data. The results confirm this hypothesis.

5. Servo-mechanisms. H. L. HAZEN, *Massachusetts Institute of Technology.* (Invited paper.)—The paper has two parts, the first relating to the theory of servo-mechanisms in general, the second to the design and test of a particular high-performance servo-mechanism. In the first part, characteristics of three types, the relay type, the definite-correction type, and the continuous-control type, are examined quantitatively, considering such factors as inertia, the nature and amount of damping, time delay in the application of restoring forces, inactive zone, nature of control for the restoring forces, etc., and their effects on oscillation or hunting, and error of following. A figure of merit for the continuous-control type is developed as a guide for comparison and design. In the second part, the design of a non-oscillating, fast-following servo-mechanism is described, together with calculations and test results on its performance. This servo, which uses light from a photoelectric cell as input and delivers about 75 watts output in the form of mechanical rotation, substantially completes its response to a change in input variation within 0.05 second. Methods of improving the performance are suggested.

6. The Inherent Power Factor of Air Condensers and the Limits of Power-Factor Bridge Measurements. J. C. BALSBAUGH, *Department of Electric Engineering, Massachusetts Institute of Technology. (Introduced by W. O. Severinghaus.)*—In general, the use of a bridge for the power-factor measurement of a dielectric will not give an absolute power-factor measurement of the dielectric, but will indicate the difference in the power factors of the dielectric and of a reference condenser. Tests have shown that air condensers may have quite significant inherent power factors, the magnitude of which is determined principally by the kind of metal surfaces, the nature of the surfaces, the air pressure or degree of evacuation and the voltage gradient. A method has been developed which will permit the evaluation of the inherent power factors of air condensers from a series of test measurements. A bridge has been developed which will permit power-factor measurements, with an accuracy expressed in power factor of 10^{-6}, to be made on small oil samples. This bridge is a modified Schering type having a single shield circuit and with capacitance balancing between bridge and shield circuits so as to minimize the effect of a shield unbalance on the measured bridge values. The measured bridge power factors may be expressed in terms of differences in absolute capacitances of the measuring capacitances.

7. Experiments Relating to the Study of the Electrical Resistivity of the Earth's Crust at Depth. L. B. SLICHTER, *Massachusetts Institute of Technology.*—It is known that knowledge of the surface potentials induced in a horizontally uniform flat earth by a direct-current flow from a point electrode uniquely determines the unknown variation of the electrical resistivity with depth. The present experiments were undertaken to obtain experience in the technique of mapping electrical potentials on a large scale on the surface of the earth. An earth current flow was established between a grounded electrode at Clinton, Massachusetts and a second electrode, about 50 km away, near Boston. A six kilowatt commutated d.c. source, and power line facilities kindly provided by the New England Power Association and the Edison Electric Illuminating Company of Boston were used. The ground potentials produced by this flow were systematically measured in an area about 80 km in diameter centering at Clinton, with the furthest point 140 km away, at Pittsfield, Massachusetts. The New England Telephone and Telegraph Company generously provided the circuit facilities for measuring these potentials. The resulting potential map is shown, and the experimental problem discussed.

8. Attempts to Produce Relief Patterns by the Direct Electrographic Effect. ELIZABETH WILCOX AND P. H. CARR, *Iowa State College, Ames, Iowa.*—The fact that the surface of many metals which has been exposed to cathode rays is rendered less active chemically toward corrosive vapors is termed the direct electrographic effect. Attempts to take advantage of this effect to produce a pattern in relief have been only partially successful. In these experiments a portion of the metal surface is protected by means of a stencil during the exposure in air to 100 kilovolt cathode rays. An attempt is then made to develop the image of the stencil on the metal by use of a suitable reagent and directly or indirectly build the image in relief. Results confirm previous reports that water vapor is helpful if not essential to satisfactory development. On silver, copper, tin, zinc, lead, brass and bismuth the inorganic materials investigated prove better developers than organic materials. The image was not rendered in relief by electroplating directly after exposure on any material except carbon. Attempts to produce the relief by first developing in the ordinary way and then electroplating failed except when anthraquinone was the developer. Cadmium vapor as a developer produces images of low relief directly.

9. The Effect of Low Speed Electrons on Silver and Bismuth. J. B. PHILIPSON AND P. H. CARR, *Iowa State College, Ames, Iowa.*—Silver and bismuth films sputtered on glass in a residual atmosphere of air have been exposed to beams of electrons having speeds of less than 100 equivalent volts. This produces a latent impression or image on the film which can be developed in vapors as described in R.S.I. **1**, 711 (1930). Results show that bismuth films developed in the air of the laboratory

become very insensitive to electrons with speeds less than 25 equivalent volts, and records below this speed were not obtained. Silver films show a decrease in sensitivity with a decrease in speed of the electrons, but records have been obtained at speeds of 7 equivalent volts when iodine vapor was used as the developing agent. Records on silver films may be satisfactorily developed in a liquid developer consisting of iodine dissolved in ethyl alcohol. Experiments performed to discover the nature of the effect indicate that in the case of silver the action of the electrons is to render the metal less active chemically.

10. A Convenient Proton Source. E. S. LAMAR AND OVERTON LUHR, *Massachusetts Institute of Technology.*—In this investigation directed toward the development of a convenient proton source, a low voltage arc was employed of the type described by Langmuir and Jones (Phys. Rev. **31**, 357 (1928)). The arc was maintained in hydrogen at pressures ranging from 0.1 to 0.5 mm of Hg, between an axial filament and two anode end plates. Ions were drawn out through a slit in the side of a coaxial metal cylinder which was maintained negative with respect to the anode. The relative numbers of the different types of these ions were studied as a function of the negative voltage on the cylinder by means of a mass spectrograph. The proton percentage increased with increasing negative voltages reaching, at 100 volts, between 80 and 90 percent. The above proportions were unchanged when helium was mixed with the gas and no helium ions were observed. The arc current was always one ampere and the arc drop about 25 volts. Under these conditions the total ion current to the cylinder was 150 milliamperes or 3 milliamperes per square centimeter. It is believed that the hydrogen is dissociated by the neutralized positive ions which rebound from the surface of the cylinder with a large fraction of their initial kinetic energy.

11. Acceleration of Ions. J. W. BEAMS AND L. B. SNODDY, *University of Virginia.*—The method of accelerating electrons (Phys. Rev. **44**, 784 (1933)) has been applied to the acceleration of protons. Essentially the method consists in making the electric field move, in effect, with approximately the same speed as the ion. Hence, the ion acquires a final velocity corresponding to a voltage many times that originally applied. The accelerating field was applied, by means of an artificial transmission line, in succession between pairs of a series of short cylindrical electrodes mounted 10 cm apart in an evacuated glass tube. The velocity of the ions was measured by their deflection in a magnetic field. In a nine electrode tube the increase in velocity of the protons corresponded to six times the maximum applied potential of 15,000 volts. The ion "blast" may be accurately timed with the expansion of a Wilson cloud chamber. The method should also serve as an ion selector. It is believed that with a new tube now under construction ion velocities corresponding to several million volts can be obtained.

12. Negative Ion Formation by Electron Attachment. NORRIS E. BRADBURY (*National Research Fellow*), *Massachusetts Institute of Technology.*—The formation of negative ions by electron attachment to neutral gas molecules has been studied by a direct method. This method permits the analysis of a mixed current stream of ions and electrons in such a manner that the rate of decrease of free electrons passing through the gas may be directly measured. A modification of the apparatus permits a simultaneous determination of the electronic mobility. The fraction of electronic impacts resulting in capture or the capture cross section may then be calculated as a function of the electronic energy. Measurements in hydrogen, nitrogen and carbon monoxide show these gases to have no electron affinity. Oxygen has a capture cross section which varies widely with electronic energy, the first excited state at 1.62 volts playing an important part. Ammonia shows no electron attachment below electronic energies of approximately three volts, but forms negative ions rapidly at higher electronic energies. The measurements are being continued for other gases, particularly nitric oxide.

13. An Investigation of an Alternating-Current Method of Determining Critical Potentials. R. W. HICKMAN, *Harvard University.*—An investigation of an alternating-current bridge method of measuring critical potentials in a vapor has been made. A tube containing mercury vapor and of similar structure to that used in the classical experiments of Franck and Hertz was used. It was placed in one arm of an impedance bridge and so arranged that a small alternating voltage in series with a small direct retarding voltage existed between the accelerating grid and collecting plate. Measurements of the equivalent variational plate resistance and reactance were made as the steady accelerating voltage was varied, the accelerating grid and the cathode being at the same alternating potential. Plots of the variational resistance or reactance, or the bridge balancing resistance or capacitance as a function of the accelerating voltage showed the typical irregularities usually found in the current-voltage plots to be enhanced. The positions of the irregularities were found to be dependent on the amplitude of the applied alternating voltage and independent of the frequency within the range investigated. Results indicated that the resolving power of the method varies inversely with the amplitude of applied alternating voltage. The effect of vapor pressure on the position and evolution of the irregularities were also studied.

14. Scattering of Molecular Rays in Gases. R. M. ZABEL (*National Research Fellow*), *Massachusetts Institute of Technology.*—The scattering of beams of hydrogen molecules and helium atoms in mercury vapor has been investigated in the region from 5° to 70° as measured from the original beam. The maximum found in the scattering curve of helium in mercury by Knauer (Zeits. f. Physik **80**, 80 (1933)) was not observed in spite of the higher intensity in the scattered beam. The results obtained for helium are satisfactorily explained on the basis of the quantum theory of scattering of rigid spheres as developed by Massey and Mohr (Proc. Roy. Soc. **141**, 435 (1933)). A simple approximation to this theory which predicts

large scattering at small angles and classical scattering at large angles (see Fig. 2 of their article) is sufficient to predict the results obtained. The scattering of hydrogen molecules shows some deviation from the results predicted by the same theory. In the case of both hydrogen and helium it is necessary to assume collision radii several times larger than the corresponding kinetic theory values in order to obtain the best agreement between the calculated and observed curves.

15. Electron Scattering in Mercury Vapor. A. P. GAGGE, *Yale University.*—The magnetic deflection method has been applied to the study of electron scattering in mercury vapor. In the large angle range (150°–180°) for electrons of energy between 4 and 100 volts the elastic scattering curves reveal a new maximum which in each case studied precedes a minimum at 180°. As the electron energy is increased, this wide angle maximum moves toward the 180° region. At about 25 volts this maximum approaches the nearest to 180° and the backward scattering for 180° is the greatest for this voltage also. Below 25 volts the inelastic scattering curves for 6.7 volt loss have a maximum at about 90°, which increases in magnitude as the electron energy is decreased. Above 25 volts these inelastic curves develop with increasing energy into their corresponding elastic shapes as has been shown earlier by Mohr and Nicoll (Proc. Roy. Soc. **A138**, 229–244 (1931)).

16. Distintegration of Sputtered Deposits. E. A. JOHNSON* AND LOUIS HARRIS,** *Massachusetts Institute of Technology.*—If a slit or edge is interposed between a receiver and cathode, the sputtered deposit shows an anomalous pattern. The patterns are shown to be due to a disintegration of the sputtered deposit by electrons and by neutralized positive ions of the sputtering gas reflected from the cathode. The sputtered particles are scattered by the gas while the neutral gas atoms leave the cathode normally and are not scattered.

17. Evaporation Technique for Highly Refractory Substances. H. M. O'BRYAN, *Massachusetts Institute of Technology.*—Platinum, boron, quartz and other substances which cannot be readily evaporated by thermal contact with a hot filament have been evaporated by electron bombardment of a crucible. Substances which cannot be obtained in wire form or which alloy with a hot filament can be evaporated from a tantalum or graphite crucible. A tungsten filament, usually at less than 2000°C, surrounding the crucible supplies electrons at 4000 volts. Molybdenum has been evaporated at about 2600°C from a graphite crucible. Radiation losses necessitate a crucible of small dimensions. A disadvantage of this arrangement is that a pressure of 10^{-5} mm is required while the common method needs only a "black" vacuum of about 10^{-3} mm. Shields of sheet metal diminish radiation losses and prevent deposition on the glass vacuum container. This method has been used in the preparation of surfaces for excitation potentials (Skinner, Proc. Roy. Soc. **A140**, 277 (1933)) and recently for x-ray targets and small mirrors.

18. Precise Measurements of Dispersion in Nitrogen. CLARENCE E. BENNETT, *Massachusetts Institute of Technology.*—A new and much improved displacement interferometer has been built after the general plan of the one previously described (Phys. Rev. **37**, 263 (1931)), to measure simultaneously refractive index and dispersion constants for a gas over a range of pressures above atmospheric. It is constructed as a single unit on a steel I-beam framework 10 feet long and 2 feet wide. The whole apparatus is supported on automobile inner tubes which very effectively eliminate vibration. The features include a G.E. S-1 sunlamp light-source, aluminum mirrors, a carefully calibrated micrometer screw of high quality, and an improved temperature control. Observations made on nitrogen at four wave-lengths, 5780A, 5461A, 4359A, and 4047A, corresponding to pressure runs up to 14 atmospheres measured by a Keyes dead-weight gauge, at temperatures of 50°C, 30°C, and 0°C, give readings that are linear with pressure to such a degree that the Cauchy dispersion constants can be expressed to four significant figures. The results for N.T.P. conditions are: $A_0-1=0.0002932$, and $B_0=1.637\times10^{-14}$. Since A_0-1 is the extrapolated infinite wave-length refractive index, a dielectric constant $\epsilon=1.0005864$ is thus predicted. This agrees with published determinations. The refractive index at any wave-length can be computed from these results, and values so calculated are found to agree with values found by other methods which are not suitable for direct extrapolation to infinite wave-length, and which are not likely to be so accurate.

19. Preliminary Results on the Analysis of the Arc Spectrum of Osmium. WALTER ALBERTSON, *Massachusetts Institute of Technology.*—1050 arc lines of osmium are found to result from combinations between 136 terms. The list includes most of the strong lines and nearly all the underwater spark lines. (Meggers and Laporte, Phys. Rev., Oct. 1926.) The classification was aided greatly by the mechanical interval recorder. (G. R. Harrison, R.S.I., Dec. 1932 and Nov. 1933.) The normal state of the osmium atom is $s^2d^6\,{}^5D_4$; ${}^5F_5$, the lowest term of the sd^7 configuration, is at 5144 cm^{-1}. Electron configurations and multiplets show such extreme overlapping that, except for a few cases, the configuration and multiplet assignments will be meaningless. The two low multiplets, $s^2d^6\,{}^5D$ and $sd^7\,{}^5F$, are partially inverted, the order of $J=2$, 3 being interchanged in each case. A feature of the array is the large number of combinations between terms. The selection rules for L and S are so completely broken down that nearly all the combinations allowed by the J selection rule have been found. New wave-length measurements are to be made using a grating having a dispersion of 0.4A/mm in the 2nd order.

* Round Hill Research Division, Department of Electrical Engineering.

** Research Laboratory, Department of Physical Chemistry.

20. Arc and Spark Spectra of Cerium. GEORGE R. HARRISON AND WALTER ALBERTSON, *Massachusetts Institute of Technology.*—Application of the mechanical interval sorter (Harrison, R.S.I., Dec. 1932 and Nov. 1933) to King's lists of Ce I and Ce II lines reveals numerous coincidences which interrelate to form quadratic arrays. The profusion of lines makes guarding against accidental coincidences imperative, and trustworthy application of the combination principle requires more accurate wave-lengths than are available. Accordingly, we have photographed the cerium arc between 2000–5000A with a 35 ft. grating, dispersion 0.4A/mm, and are extending these measurements into the infrared. The accuracy of our wave-lengths appears limited only by the accuracy of the tertiary iron standards, results being consistent to 0.002A, which renders highly probable the meaningfulness of quadratic arrays obtained with them. Theory indicates that the strongest Ce I lines lie in the infrared, explaining previous failures to get absorption lines in the underwater spark. The recent partial analysis of Ce I by Karlson (Zeits. f. Physik **85**, 482 (1933)) shows internal deviations, which, considering the number of lines from which he made selections in forming an array, throws doubt on a number of his terms.

21. The Spectrum of Mo V. M. W. TRAWICK, *Cornell University.*—The vacuum spark spectrum of molybdenum has been photographed in the region below 2500A. The transitions $4d5p\ ^3(F^0D^0P^0)$ into $4d^2\ ^3(FP)$ and $4d5s\ ^3D$ in Mo V have been assigned to lines identified in part by extension of the Sr I-like isoelectronic sequence. Some of the transitions involving terms arising from the $4d5d$, $4d4f$, and $4d6s$ configurations have also been determined. This work has been facilitated by the use of unpublished data kindly supplied by Professor R. J. Lang on the spectrum of Cb IV.

22. Wave-Length Standards in the Extreme Ultraviolet. J. C. BOYCE, *Massachusetts Institute of Technology.*—With the two-meter focus normal incidence vacuum spectrograph previously described (Compton and Boyce, Phys. Rev. **40**, 1038 (1932)), the spectra from electrodeless discharges in neon, argon, krypton, and xenon have been photographed. In the course of the measurement of the neon plates, which extend from about λ200 to λ2530, a number of impurity lines of carbon, nitrogen, and oxygen were measured in their higher orders against the first order of the iron arc lines in the region λ2250 to λ2530. These lines are among those previously established as standards by Bowen and Ingram (Phys. Rev. **28**, 444 (1926); **29**, 231 (1927)) and more recently remeasured by Edlen (Zeits. f. Physik **85**, 85 (1933)). Present results from several exposures on each of two different plates show excellent agreement with the measurements of Edlen below λ800 but definitely depart by small amounts from the results of the previous investigators for N I λ1134 and λ1200 and C III λ1175. Suggested revised values for these wave-lengths will be published when more plates have been measured. Meanwhile for standards in this region it is suggested that use be made of second orders of Edlen's at shorter wave-lengths and also of Paschen's calculated values for the Lyman series of hydrogen (Preuss. Akad. Wiss. Berlin, Ber. 1929, p. 662).

23. The Absorption Spectrum of the Iodine Atom. J. H. MCLEOD, *Harvard University.*—The absorption spectrum of atomic iodine has been investigated in the spectral range λ2100A to λ1350A. The iodine was dissociated by heating to 1000°C. Light from a hydrogen discharge tube was passed through the heated iodine and recorded on the photographic plate of a vacuum grating spectrograph. Two spectral lines 1830A and 1783A were recorded as absorption lines. For the investigation of the spectral region below λ1650A the hydrogen discharge tube was replaced by a discharge tube containing iodine. When the light from the discharge in iodine was passed through the heated iodine the following lines were found to be weakened by absorption: 1830, 1783, 1642, 1618, 1583 and 1515A. In addition there was some indication that the lines 1507, 1459, 1453 and 1383A were somewhat absorbed by the iodine atoms.

24. Hyperfine Structure of Fluorine. F. W. BROWN, *University of Illinois. (Introduced by J. H. Bartlett, Jr.)*—The theory of Breit and Doermann (Phys. Rev. **36**, 1732 (1930)) has been extended and applied to the case of fluorine. The wave functions for the $2p^4\,3s$ and $2p^4\,3p$ configurations were found by the self-consistent field method. Assuming Russell-Saunders coupling, and with the experimental results of Campbell (Zeits. f. Physik **84**, 393 (1933)), the magnetic moment of the fluorine nucleus has been calculated. Fairly consistent results are obtained, and the value of the nuclear magnetic moment is found to be about 3 nuclear magnetons.

25. A Modification of Levi-Civita's Wave Equation. BANESH HOFFMANN, *University of Rochester.*—Levi-Civita has recently proposed a modification of Schrödinger's relativistic wave equation in which the ψ-function is replaced by a four-vector ψ^α, and electron spin is introduced by the arbitrary inclusion of "coupling" terms of the form $Ce^{abcd}g_{ae}F_{cd}\psi_b$, where F is the electromagnetic six-vector and C is a constant. The value of C is chosen as $(\pi e/ch)$ in order to give the correct magnitude for the spin. The author has considered ψ^α as being a degenerate projective vector having $\psi_0=0$; the simplest projective vector wave equation involving only four equations is

$$\gamma^{\alpha\beta}\psi_{\gamma;\,\alpha\beta}=\varphi_\gamma(\gamma^{\alpha\beta}\psi_{\epsilon;\,\alpha\beta})_{\epsilon=0},$$

with the notation of the projective relativity theory. This equation automatically contains coupling terms equivalent to those introduced by Levi-Civita, and these terms automatically have the correct coefficients. A further set of terms contained in it is negligible unless $r\sim10^{-13}$ cm or less. Apart from the advantages enumerated, the wave equation suffers from the same disadvantages as does that due to Levi-Civita.

26. Introduction to the Theory of the Positron.* G. E. UHLENBECK, *University of Michigan.*—First the considerations of Dirac will be shortly reviewed which led necessarily and almost uniquely to his relativistic wave equation for the electron. The great success of this theory is well known; it has explained the spin of the electron and from it follow the Sommerfeld fine structure formula and the Klein-Nishina formula for the scattering of free electrons which are both experimentally so well confirmed. But with the formalism of the theory were intimately related the difficulties connected with the states of negative kinetic energy. Transitions to these states are namely possible (most simply seen in an example given by Klein), in contrast with experience. Dirac has tried to overcome these difficulties by assuming that almost all negative states were "filled" in the sense of the Pauli principle. An unoccupied negative state or "hole" will behave like a particle of positive charge and positive mass which is, as one could show, the same as that of the electron. It could be destroyed by combination with an ordinary electron, whereby the energy connected with the masses ($=2mc^2=10^6$ volt) would be transformed into radiation. These ideas have now been beautifully supported by the discovery of the positron. And although this way of interpreting the Dirac equations is difficult to accept quite literally, it can be used already as Oppenheimer and Plesset have shown, to derive quantitative results for the probability of formation of an electron-positron pair by absorption of hard γ-rays near a heavy nucleus. In the same way one can calculate the probability of destruction of a positron by combination with an electron. Thus one can understand certain puzzling features in the absorption and scattering of γ-rays. One may hope therefore that further experimental and theoretical work will show the essential consistency of the Dirac theory with the experimental facts.

27. The Positron.* CARL D. ANDERSON, *California Institute of Technology.*—Positrons were first reported in September, 1932, to be present among the secondary particles of the cosmic radiation. During the present year the generation of them by hard gamma-rays, and by bombardment of various light elements by alpha-particles has been reported. The various researches which have been carried out in this country and abroad on their production and properties will be discussed, including new experiments on both the cosmic-ray and Th C″ gamma-ray effects.

28. The Theory of the Electron and Positive.* J. R. OPPENHEIMER, *University of California, Berkeley, Calif.*—The most recent theoretical results on the production of positives by a beam of gamma-rays, by internal conversion of gamma-rays, and by electron impact are compared with experiment. In general the theory agrees well with observation for radioactive gamma-rays and those produced in nuclear disintegration, but fails entirely for gamma-rays of very high energy, and gives no understanding of the primary processes in cosmic-ray absorption. It also fails to explain the positives observed in disintegrating aluminum. The limitations of the theory which are indicated by these failures are formulated, and it is shown that within these limits it is possible to give a complete and consistent formalism for describing the state of a system containing electrons and positives, and computing the energy levels and transition probabilities of the system. Where the original Dirac theory gave results in agreement with experiment, the present formalism gives the same results; and the probabilities of pair production agree within wide limits with observation. The physical interpretation of the formalism is discussed, with particular reference to the question of the localizability of the electron, and it is shown that no complete spaciotemporal description of the system in terms of particles is possible. The effect of the pairs upon the electromagnetic field of particles is discussed, and it is shown in particular that the pairs contribute to the forces between two point charges. For large distances of separation between the particles the field is Coulomb; the difference between the "effective" charge which gives these forces and the "true" charge on the particles cannot be unambiguously computed, and is in our opinion not observable; but for smaller distances of approach of the particles, deviations from the Coulomb law of force appear, which are of the order of 1 percent for distances of the order of the Compton wave-length. These may be computed, and should ultimately be observable.

29. Industrial Physics.* PAUL D. FOOTE, *Gulf Research Laboratory*, Address of the President of The American Physical Society.

30. Current Progress in X-Ray Physics.* D. L. WEBSTER, *Stanford University*, Address of the Retiring Vice-President of Section B—A.A.A.S.

31. Hydrogen in Palladium.* JAMES FRANCK, *formerly of the University of Göttingen.*

32. Vibrational Analysis of the Green BaCl Bands. ALLAN E. PARKER, *Yale University and Columbia University.*—A study has been made of the green bands of BaCl by using as source an arc in a hydrogen atmosphere between a nickel anode and a copper cup containing $BaCl_2$. The vibrational analysis shows the existence of the $\Delta v=0, \pm 1$ sequences, extending from λ5056 to λ5322. The bands are attributed, as are the green BaF bands, to a $^2\Pi-{}^2\Sigma$ transition, the four head-forming branches being the Q_1, R_1, Q_2 and R_2 branches. The major portion of the bands observed arise from the molecule $Ba^{138}\,Cl^{35}$ and the remainder can be ascribed to $Ba^{138}\,Cl^{37}$. The Q_1 and Q_2

* Invited paper.

heads are given by the following expressions:

$$\nu_A = 19{,}062.9 + [280.2(v'+\tfrac{1}{2}) - 0.79(v'+\tfrac{1}{2})^2] - [278.4(v''+\tfrac{1}{2}) - 0.80(v''+\tfrac{1}{2})^2],$$

$$\nu_A = 19{,}450.1 + [285.0(v'+\tfrac{1}{2}) - 0.79(v'+\tfrac{1}{2})^2] - [280.5(v''+\tfrac{1}{2}) - 0.80(v''+\tfrac{1}{2})^2].$$

The bands at low values of v have sharp heads and degrade to the red. At high v values they become somewhat diffuse due to B' becoming very approximately equal to B''.

33. On Abnormal Rotational Temperature of Band Spectra. O. OLDENBERG, *Harvard University.*—(1) The abnormal intensity distribution of rotational quanta ("rotational temperature" of excited molecules much higher than the real temperature) has been interpreted in previous papers. It was pointed out that polyatomic molecules dissociating by excess *vibration* of certain modes might produce fragments with abnormal *rotation*, the energy of which is determined by the relation between potential curves and not by the temperature. This hypothetical statement is confirmed by the intensity distribution of the OH band in the discharge through water vapor. Absorption spectra of OH, taken in connection with another investigation, revealed a contrast with the emission spectra. In emission, lines of high rotation appear, although according to the temperature molecules with the corresponding energy ought to be present only with a relative concentration of 10^{-23}. For the discharge through water vapor it is known by independent arguments that the excited OH molecules are largely produced by the dissociation of excited water molecules. (2) The effect of foreign gases reducing the abnormal rotation by collisions is being studied. Helium shows a pronounced effect of this kind.

34. Pressure Coefficients of Acoustic Velocity for Nine Organic Liquids. J. CHESTER SWANSON, *Duke University*, with J. C. HUBBARD, *The Johns Hopkins University.*—From thermodynamic considerations it seemed desirable to study the change in velocity of a compressional wave in liquids at different pressures. For this purpose a supersonic interferometer was designed by Swanson (R.S.I. **4**, 603 (1933)) the essential feature of which was the controlled movement of a reflector within a chamber containing the liquid under pressure, without introducing a change of pressure by the movement. Pressures up to about 300 kg/cm² were used and a quartz crystal was driven at about 200 kc by a Hartley oscillator as the source of the high frequency waves. The liquids examined show an increase of velocity with increasing pressure of the order predicted by thermodynamic theory. The liquids studied were: ethyl bromide, carbon tetrachloride, chloroform, carbon disulphide, benzene, toluene, aniline, ethyl ether and pentane. The temperature was that of the room which never varied over two degrees from 23°C for the whole series of readings and generally much less than one degree for any run of one liquid. The average variation in velocity (M/sec.) per 100 kg/cm² over the pressure range of 300 kg/cm² for each liquid is given below:

C_2H_5Br, 38.3;	CCl_4, 45.3;	$CHCl_3$, 33.0;
CS_2, 44.0;	C_6H_6, 42.0;	C_7H_8, 44.7;
$C_6H_5NH_2$, 35.0;	$C_2H_5OC_2H_5$, 87.3;	C_5H_{12}, 92.0.

35. The Magnetostrictive Oscillation of Chladni Plates. R. C. COLWELL AND E. A. BRYANT, *West Virginia University.*—A rod of nickel placed in the grid coil of a vacuum tube circuit will be set in vibration at radiofrequencies. A Chladni plate balanced at the center on top of the nickel rod will also be set in vibration by mechanical impact. The radio vibrations measured upon a wave meter were 14,000 per second; the calculated vibrations of the plate were 3500, 2800, etc. At these very high frequencies many new sand patterns may be formed upon square or circular plates. A few of these will be shown.

36. A Magnetostrictive Phonograph Reproducer with Demonstrations. S. A. BUCKINGHAM, *Harvard University.* (*Introduced by G. W. Pierce.*)—A phonograph reproducer has been constructed employing the principle of magnetostriction. The magnetostrictive element is a bimetallic strip of nickel and phosphorbronze. It is found that the length of the strip must be such that the lowest natural mode of transverse vibration is well above the highest frequency to be reproduced. When this condition is observed the fidelity of the reproducer compares favorably with that of the conventional magnetic type. In the present state of development, the output of the reproducer is approximately 30 decibels below that of the conventional reproducer and the stiffness exceeds that of the magnetic type. A working model of the instrument will be demonstrated.

37. Maintenance of the Earth's Electrical Charge by Electrical Rectification in the Earth's Lower Atmosphere. ROSS GUNN, *Naval Research Laboratory, Washington, D. C.*—The relation of the current density to the impressed electric field intensity is worked out for the earth's atmosphere, taking account of space charge. It is shown that the departures from Ohm's law are such that electricity is transferred outward from the earth with greater ease than inward. Thus the large random alternating electric fields of the type observed during storms will result in a net upward transfer of electricity which may leak off over a large area and contribute to, or account for the entire normal or fair weather electric field. The fair weather field is downward and hence, with given field intensities, the electrical discharge of the earth is more difficult than is the process of replenishment. The fundamental action of the atmosphere as an electrical rectifier is emphasized and it is noted that as a result of this property only *radially inward* atmospheric electric *fields* can be *stable.*

38. Progress Report on Kennelly-Heaviside Layer Measurements. HARRY R. MIMNO AND PAO H. WANG, *Harvard University.*—Automatic devices, which make continuous daily records of the equivalent heights of the Kennelly-Heaviside layers, have been described in a number of previous reports from this laboratory and from

other laboratories (Phys. Rev. **41**, 395 (1932); I.R.E. **21**, 529 (1933)). The present paper summarizes some of the results obtained during more than 6000 hours of operation in 1933. During a large portion of this period we recorded two frequencies simultaneously. We have plotted curves which indicate the degree of correlation between magnetic disturbances and layer heights at Cambridge. We have also examined the effect of magnetic disturbances in determining the hour at which penetration occurs. From the recorder measurements, it is possible to make quantitative estimates of the normal rates of ionization and recombination. Abnormal early-morning E layers, previously reported, were again noted on many occasions. Other observers have suggested that these may result from magnetic disturbances, or from local storm conditions. The examination of more than 20 cases does not yet provide experimental support for either view. A third recorder has been constructed. In the newest design, a revolving drum carries a roll of sensitized paper across the beam produced by a stationary glow lamp. The paper strip unrolls automatically as the drum rotates. Simultaneous recorder measurements have been made at different geographic points.

39. The Interpretation of X-Ray Diffraction Curves of a Fluid from a Gaseous to a Liquid State. N. S. GINGRICH AND B. E. WARREN, *Massachusetts Institute of Technology.* —An attempt has been made to find an expression for the intensity of scattering of x-rays in a monatomic fluid as a function of density, sufficiently general to hold continuously from the liquid to the rarefied gas. It is found that a very simple approximate model leads to scattering curves in good qualitative agreement with experiment. It is assumed that, surrounding any one molecule in the liquid, there is a continuous distribution of molecules beyond a distance b, roughly the intermolecular distance, and a concentration of molecules at a slightly smaller distance a. It is further assumed that in passing to the gaseous state, the relative density distribution remains the same, while the actual density decreases. The intensity of scattering from this model is calculated by the method of Zernicke and Prins, and a set of scattering curves obtained for several densities of fluid. In the limiting case of small density, the curves are typical of those for gas scattering, and for maximum density, typical of liquid scattering, with continuous transition between. Although the model used is only a rough approximation for ether, there is still a good qualitative agreement between the set of curves obtained, and the experimental curves of G. W. Stewart for ether at various specific volumes.

40. The Atomic Arrangement in Vitreous SiO_2 and GeO_2. B. E. WARREN, *Massachusetts Institute of Technology.*—X-ray scattering curves for vitreous SiO_2 and GeO_2 have been made with considerably improved experimental conditions. The radiation Cu $K\alpha$ is monochromatized by reflection from a NaCl crystal, and passed through a thin sample of the glass in a cylindrical camera. The camera is evacuated to eliminate air scattering. By these precautions the troublesome background is largely eliminated. SiO_2 shows a single strong peak at $(\sin\theta)/\lambda = 0.118$ and GeO_2 a strong peak at $(\sin\theta)/\lambda = 0.118$ and a weaker peak at 0.194. The atomic arrangement in the two glasses is the same; each cation is tetrahedrally surrounded by 4 oxygens and each oxygen is shared between two tetrahedral groups. The orientation about the direction of bonding is random, and the resulting network nonrepeating. It is this feature which distinguishes the glass state from the crystalline state. Scattering curves for SiO_2 and GeO_2 calculated for such an atomic arrangement are in very good agreement with the experimental curves. Within the experimental error the interatomic distances in the glass are found to be the same as in the crystalline modifications.

41. Rotation of the Plane of Polarization of Beam of X-Rays. RONALD LYMAN MCFARLAN, *Harvard University.* —The question, as to whether or not the plane of polarization of a plane polarized beam of x-rays is rotated on passing through a quartz crystal in a direction parallel to the optic axis, is investigated experimentally. Quartz crystals 0.75 mm thick are observed to rotate the plane of polarization through an angle of about 2° 30′. The investigation of thicker crystals indicates that the angle of rotation is not proportional to the thickness of the crystal. This is ascribed to the effects of double scattering and to the linear absorption of the beam. Arguments are advanced to show that the rotation is almost entirely due to the x-rays scattered according to the Bragg law in the crystal.

42. The Measurement of Absolute X-Ray Intensities and Absolute Sensitivity of X-Ray Film with a Geiger-Mueller Counter. GORDON L. LOCHER, *National Research Fellow, Bartol Research Foundation of the Franklin Institute,* and DONALD P. LEGALLEY, *The Pennsylvania State College.* —A special G-M counter of small dimensions has been constructed for measuring absolute intensities of x-rays of known wave-lengths. The rays pass through the counter in such a way that only electrons set free in the *gas* are counted. The absorbing gas column is krypton at 6 cm pressure, 1 cm thick, and 0.0062 cm^2 in cross section. For fluorescent ZrK x-rays, the absorption is 3.84 percent of the beam, as calculated for an energy distribution of the Zr radiation $K\alpha_1 : K\alpha_2 : K\beta = 0.562 : 0.281 : 0.157$, by using absorption coefficients as computed from Richtmyer and Warburton's formula. A "standard beam" of fluorescent Zr rays from a special x-ray tube has been calibrated with the counter; its intensity is 1.66×10^4 quanta cm^{-2} $sec.^{-1}$, 57.3 cm from the fluorescer, of which 0.75 cm^2 area is exposed. With this beam, the minimum exposure of x-ray film for detectable blackening is found to be 0.75×10^6 quanta cm^{-2}, incident on the film, or 6.35×10^4 quanta cm^{-2}, absorbed in the emulsion of the film (Eastman Ultra-Speed Duplitized X-Ray Film). The mass absorption coefficients of the film, celluloid, and emulsion, for the Zr rays are found, respectively, to be 8.81 cm^{-1}, 1.88 cm^{-1} and 18.4 cm^{-1}. A time-blackening densitometer curve has been obtained for the same film and radiation, for exposures between 2.5 min. and 60 min.; after 10 minutes (10^7 quanta cm^{-2}), the blackening (B) and the time (t) are

related by the equation: $\log B = 0.54 \log t + \log 3.5$. The average number of quanta required to blacken a grain is found by means of photomicrographs of the blackening of a spot exposed for 15 minutes (1.5×10^7 quanta cm^{-2}).

43. Soft X-Rays and Energy States of the Conduction Electron. H. W. B. SKINNER AND H. M. O'BRYAN, *Massachusetts Institute of Technology.*—The emission spectra of a number of the light metallic elements have been photographed in the region from 40A to 600A. The target surfaces were prepared by distillation of the metal in vacuo and the results are believed to be characteristic of the pure polycrystalline solid. The widths of the lines observed are due to the initial state of the transition and agree with those computed from the Sommerfeld electron gas theory. The shape of the beryllium line agrees in general with the transition probabilities computed by Houstoun. All elements having two or more conducting electrons per atom show departures from the simplest electron gas theory. Beryllium, magnesium and aluminum indicate that certain velocities of the conduction electron are forbidden within the lattice. The de Broglie wave-lengths associated with these forbidden velocities are given by Bragg's law. Silicon shows a distinctly different type of distribution for the energy levels, resembling more what is expected with bound electrons. Ordinary x-ray selection rules do not hold for these transitions. A dirty beryllium target gave a spectrum consisting of the K line superimposed on a background of continuous radiation.

44. Electron Diffraction and the Imperfection of Crystal Surfaces. L. H. GERMER, *Bell Telephone Laboratories, Inc., New York.*—Bragg reflections are obtained by scattering fast electrons (0.05A) from etched surfaces of metallic single crystals. The surfaces studied are an iron (100) face, a nickel (111) face and a tungsten (110) face. Each reflection occurs accurately at the calculated position, with no displacement due to refraction. A reflection occurs, however, even when the glancing angle of the primary beam differs considerably from the Bragg value—by over 1° in some cases. The accuracy with which a glancing angle must be adjusted is a measure of the degree of perfection of a crystal surface. Estimates are made of widths at half maximum of electron rocking curves. These estimates are 0.8° for the iron crystal, 1.5° for the nickel crystal and somewhat over 1° for the tungsten crystal. X-ray rocking curves for these same crystals are much narrower, although the observed widths vary considerably with surface conditions. It is concluded that the values obtained from the electron measurements apply to projecting surface metal only, and that *the degree of misalignment is much greater at the surfaces than within the crystals.* Furthermore, even the x-rays [Mo $K\alpha$ radiation -0.71A] are not sufficiently penetrating to yield values characteristic of these crystals.

45. Electron Diffraction by the Oxides of Nitrogen. LOUIS R. MAXWELL, V. M. MOSLEY AND LOLA S. DEMING, *Bureau of Chemistry and Soils.*—Electron diffraction has been obtained by the transmission of electrons (20–35 kv) through molecular beams of N_2O, NO_2, and N_2O_4. The molecular structures were obtained in the usual manner by utilizing the Debye interference theory for single molecules with the proper atomic scattering amplitudes, together with the inclusion of the incoherent electron scattering as formulated by Morse (Phys. Zeits. **33**, 443 (1932)) and Bewilogua (Phys. Zeits. **33**, 688 (1932)). The results can be summarized as follows: (1) Nitrous oxide; for a linear model the separation of the end atoms was found to be 2.38 ± 0.05A in exact agreement with Wierl's measurements on this structure (Ann. d. Physik **8**, 521 (1931)). Unfortunately it is impracticable to distinguish between models of the form N-O-N and N-N-O. (2) Nitrogen dioxide and nitrogen tetraoxide; photographs obtained from a mixture of N_2O_4 and NO_2 showed only one diffraction ring at $(\sin \theta/2)/\lambda = 0.455 \pm 0.01$ which disappeared when the gas was completely decomposed to NO_2. These results are compatible with a triangular NO_2 structure as definitely distinguishable from a linear model and with a $O_2N\text{-}NO_2$ form with an N-N distance of approximately 1.6A. This is within the range of the N-N distance considered possible by Hendricks from an analysis of Vegard's x-ray diffraction photographs of solid N_2O_4 (Zeits. f. Physik **70**, 699 (1931)).

46. Group Phenomena in Metal Crystals. A. GOETZ, *California Institute of Technology.*—Discussion of experimental results concerning properties of metal crystals which depend on the magnitude of the crystal. Most properties typical for a solid metal seem to be caused by large but limited groups of molecules. The "critical" size above which a property of a crystal becomes size-independent lies for the metals investigated between 10^7 and 10^9 atoms. Evidence indicating the existence and size of such groups have been obtained by the study of the magnetic properties of microscopic crystals of graphite, Bi and Sb and by the determination of the effects of foreign atoms inserted in small concentrations into macroscopic crystals. The magnitude of the groups derived for different properties but for the same crystal comes out to be the same.—There seems to be a close analogy between these group phenomena and the "swarm" effects observed in anisotropic liquids.

47. The Recrystallization of the Metallic Layers. A. B. C. ANDERSON AND A. GOETZ. *California Institute of Technology.*—Discussion of resistivity measurements of thin layers of Ag deposited on amorphous and crystalline surfaces (calcite) by evaporation in vacuo. The change of conductivity after deposit under controlled conditions is used as a measure of recrystallization. It is found that the tendency of the isolated atoms of forming aggregates is independent of the number of atoms present and that the resistance per layer decreases suddenly when a definite statistical thickness is reached. (300 atoms.) This indicates the formation of crystalline groups of definite size, 10^{-5} to 10^{-4} cm in diameter and are commensurable to the size of the primitive etch figure. The results are in agreement with certain findings of electron spectroscopy about the state of thin layers and support the hypothesis of the formation of large atomic groups in a crystal.

48. Vacuum Tube Characteristics in Relation to the Selection of Coincident Pulses. LEWIS FUSSELL, JR., *Massachusetts Institute of Technology* and THOMAS H. JOHNSON, *Bartol Research Foundation of the Franklin Institute.*—The characteristics of various vacuum tubes have been analyzed in relation to their use in the Rossi type of circuit for selecting coincident G. M. counter pulses. The ability of the circuit to discriminate between partial and total coincidences is found to depend upon the ratio of the tube impedance under normal conditions to the impedance when cut off by a pulse and the best discrimination is realized from the superamplifier pentodes such as the '57. In a test circuit containing ten '57 tubes in parallel, according to the Rossi scheme, when pulses of one volt were simultaneously applied to the grids of all tubes the added output pulse was more than one hundred times that resulting from pulses on but nine of the grids. A one stage coincidence selecting circuit with these tubes has most of the advantages of the three stage Johnson-Street circuit. However, the three-stage circuit has the ability, not possessed by a one-stage circuit, of reducing the pulse time below the actual duration of the counter discharge so that the number of "accidentals" may be reduced.

49. Directional Measurements of the Cosmic Radiation and Their Significance. THOMAS H. JOHNSON, *Bartol Research Foundation of the Franklin Institute.*—The west-east asymmetry of the cosmic radiation increases from two or three percent at 50° geomagnetic latitude to thirteen percent at the magnetic equator. This effect, as well as the latitude intensity variations found by J. Clay and A. H. Compton, are described by the Lemaitre-Vallarta theory in terms of the action of the earth's magnetic field on an electrically charged corpuscular radiation. Since the latitude effect depends upon the sum of the intensities of the positive and negative radiations whereas the asymmetry depends upon their difference, a comparison of the two effects gives a measure of the relative proportions of the two signs of charge. In the energy range from 5×10^9 to 2×10^{10} volts the corpuscular radiation is principally positive and within the present experimental accuracy it may be entirely positive though a negative component of lower intensity cannot be definitely excluded. The azimuthal asymmetry at the equator establishes the existence of corpuscular rays of higher energy than had been predicted by the latitude effect and the theory shows that the observed asymmetry is consistent with the small magnitude of the intensity variations between the equator and the 20° parallel.

50. A Possible Explanation of the Frequency Distribution of Size of Hoffmann Stösse. C. G. MONTGOMERY, *Bartol Research Foundation of the Franklin Institute.*—The cloud-chamber experiments of Blackett and Occhialini, and Locher lend weight to the idea that Hoffmann Stösse are the result of the cooperation of several atoms in the formation of groups of rays rather than a burst of rays produced by a single atom. If one assumes that every group of rays formed contains entities capable of producing other groups, then the total number of rays which will be produced in a block of material will depend upon where the first group is produced. This results in a frequency distribution of Stoss sizes which falls off hyperbolically as the size increases and has a sharp lower limit which is the size of the first group produced. Such computed distribution curves are compared with observations of Messerschmidt, and W. F. G. Swann and the author. The upper limit to the average distance between the formation of successive groups of rays is determined to be 0.3 cm in iron and 0.6 cm in lead. The production of secondaries all along the path of a primary does not enable a distribution curve of the observed type to be obtained. The mechanism here suggested contains, of course, a consequence already under experimental investigation by W. F. G. Swann and the writer, that the size of a Stoss depends upon the thickness of the material in which it originates. The theory also predicts that certain "transition" effects are to be expected.

51. Comparison of Absorption Coefficients of Different Elements for Cosmic Rays. J. C. STEARNS AND CARL HEDBERG, *University of Denver.*—The cosmic-ray energy absorbed by 6 inches of aluminum, zinc, copper and lead respectively, was determined by a double coincident Geiger-Müller counter. With the counters in a vertical plane, the number of coincidences during a twenty-four hour period was determined with the respective elements above mentioned between the counters. These tests were alternated with runs of the same type with only air between the counters. The zero of the instrument was determined by placing the counters in a horizontal plane with the same absorbing material above or below each counter as was used in the actual test and determining the number of coincidences as the distance between the counters was increased. From these data the number of accidental coincidences were estimated by extrapolation. The data indicate that the ratio of linear absorption coefficient to density decreases with increase in density. By placing the same absorbing material above and between the counters it was possible to estimate the secondaries or tertiaries produced in and penetrating the absorbing blocks. At 11,000 feet the ratio of the number of coincidences with 6 inch lead above to those with the same lead between was 1.12 while at 8000 feet it was 1.10. This ratio for 4 and 2 inches of lead was 1.20 and 1.36 respectively.

52. Further Geographic Studies of Cosmic Rays. A. H. COMPTON,[1] J. M. BENADE[2] AND P. G. LEDIG.[3]—Since our last communication, we have measured the intensity of the cosmic rays, using the standard ionization chamber method previously described, in Singapore, the Chinese coast, Japan, and Chicago, also at various points along both coasts of South America, including the Straits of Magellan. All of the observations confirm the minimum previously

[1] University of Chicago.
[2] Forman Christian College, Lahore.
[3] Carnegie Institution of Washington.

reported by Clay and ourselves near the equator. The data confirm also our earlier conclusion that this latitude effect depends upon the location relative to the magnetic rather than geographic pole. The South American data suggest, however, that a large region whose magnetic field differs from the average for that latitude, has its cosmic-ray intensity appreciably affected by the local magnetic field. Such an effect would mean that the cosmic-ray particles are appreciably bent by the magnetic field within a distance above the earth comparable with the linear dimensions of the area concerned, in this case of the order of 1000 miles. New checks on the data got by Allen Carpe on Mt. McKinley in 1932 show that at high altitudes the latitude effect continues to latitudes considerably higher than at sea-level. This shows the existence of electrically charged cosmic rays which have insufficient energy to penetrate the atmosphere.

53. Secondary Photons in Cosmic-Ray Showers. CARL D. ANDERSON AND SETH H. NEDDERMEYER, *California Institute of Technology.*—Stereoscopic observation of cloud-chamber tracks in a magnetic field of 17,000 gauss (in some cases 800 gauss) has shown the presence in positron-electron showers of *large numbers* of non-ionizing rays. A comparison between low energy positron-electron pairs and small showers produced in plates of lead by these secondary rays, and those produced by Th C″ γ-rays has shown a striking similarity in the effects of the two radiations. It is concluded therefore, that the bulk of the non-ionizing secondary rays in cosmic-ray showers are photons. To test for the presence of the ordinary neutrons, a search was made for projected carbon nuclei and nuclear disintegration products from a plate of graphite, and none were observed. The conclusion is reached that ordinary neutrons are not present in considerable numbers, at least in the energy-range where their presence would be indicated by the usual tests. In several cases a large number of positrons and electrons (in one example more than 80 distinct tracks) with energies of the order of several million electron volts occurred which could not possibly have originated except from a considerable number of separate centers, thus indicating large numbers of secondary photons. Pair production by high speed positrons or electrons does not occur frequently since in only two cases out of more than five hundred where the particles penetrated a plate of lead, were secondary positron-electron pairs observed.

54. Neutron Emission. J. R. DUNNING AND G. B. PEGRAM, *Columbia University.*—The spectrum of the neutrons emitted from a radon-Be source has been analyzed by the ranges of recoil protons. The presence of three groups of alpha particles causes the energy distribution to be more complicated than the RaF-Be disintegration studied by Chadwick (Proc. Roy. Soc. **A142**, 1). There are a large number of neutrons in the group having a maximum energy of about 4.8×10^6 e.v., and smaller groups of about 6.5×10^6 and 8.0×10^6 are also prominent, while low energy groups of the order of 0.5 to 1.5×10^6 e.v. appear to be present. A small number of neutrons have energies ending at around 12.5×10^6 e.v. and there appear to be a few neutrons having energies up to approximately 14.2×10^6 e.v. Assuming the reaction: $Be^9+He^4\rightarrow C^{12}+n$, this energy indicates a neutron mass of approximately 1.0066. Careful search for higher energy neutrons shows no evidence for their presence. From the energy distribution, two resonance levels are probably present, but a large part of the neutron emission appears to be due to penetration over the potential barrier, with accompanying gamma-ray transitions. We have failed to find the large neutron emission from a radon-zinc source, about 50 times that from RaF-Be, reported by Kirsch and Matzner (Naturwiss. **35**, 640). Our measurements indicate that the number of neutrons of energy greater than 0.5×10^6 e.v. from radon-Zn, if any, is less than 1/50 the number from radon-Be. Fruitless search for neutrons from the possible disintegration of deuterium by alpha-particle bombardment of "heavy water" has indicated that the probability of such disintegration is less than 1 to 10^7 alpha-particles.

55. Cosmical Electric Fields. W. F. G. SWANN, *Bartol Research Foundation of the Franklin Institute.*—Some time ago the writer proposed a modification of electrodynamics designed to secure a slow death of positive electricity in the earth, with the result that the corresponding negative electricity passed off to infinity as the atmospheric electric current. It is now pointed out that the space charge of this distribution creates a potential in the body from which it comes. The detailed working out of the situation leads to a potential which would become logarithmically infinite in infinite time. However, with times of such an order of magnitude as to have significant meaning in our universe, a logarithmic infinity is not a complete barrier to the attainment of a reasonably definite meaning to the story of what actually may happen. In application of the idea to stars it is possible to see how on a basis of the fundamental assumption concerned, potentials of the order of those encountered in cosmic-ray phenomena may arise.

56. An Analysis of the Double Crystal Spectrometer and the Determination of X-Ray Line Shapes. LLOYD P. SMITH, *Cornell University.*—A somewhat finer grained mathematical analysis of the action of a double crystal spectrometer has been carried out, as a result of which a method of using the instrument has been devised which makes it possible to determine the diffraction patterns of the crystals experimentally and to resolve the problem of determining the spectral energy distribution of x-ray lines.

57. The Distribution of Initial Velocities of Positive Ions from Tungsten. GEORGE J. MUELLER, *Cornell University. (Introduced by Lloyd P. Smith.)*—The distribution of initial velocities of positive ions emitted from a hot tungsten filament in vacuum has been investigated by measuring the positive ion current from a short length (0.05 cm) of filament to a coaxial cylindrical electrode against various retarding potentials. Measurements were made for eighteen different temperatures ranging from 1300°K to 3000°K. The ion current consisted of not only

tungsten ions but also those of impurities in the metal, chiefly potassium. Within experimental error, it was found that the ions were emitted with velocities distributed in accordance with Maxwell's law.

58. Precipitation Hardening and Secondary Structure. K. H. MOORE, *Rensselaer Polytechnic Institute, Troy, N. Y. (Introduced by R. A. Patterson.)*—This is a preliminary report upon the use of polarized light in the investigation of the possible connection between the phenomenon of precipitation hardening and the secondary or mosaic structure of metallic crystals. The structure of an age-hardened alloy of copper-beryllium, (2.5 percent Be), was studied by means of an improvized polarization microscope in an effort to connect the favored planes upon which the first precipitation takes place with the hypothetical net of planes making up the secondary structure of crystals. Precipitation, and coalescence of the precipitated particles, were found to take place at the grain boundaries first, as expected. After a light etch to remove polishing flow, etc., the polarization microscope, with crossed nicols, and in some cases a retarding plate, revealed nets of lines, indicating the surface traces of favored planes; these being invisible under normal or conical non-polarized illumination. The question of whether these are due to internal strain or actual presence of precipitate has not been answered as yet. The usual period of etching with the customary concentration of etchant reveals a diffuse form of the same type of structure, because of preferential etching. With crossed nicols, the nets of lines, observed after light etching, were resolved. The spacing of two rectangular nets, apparently a pattern of squares, was measured as slightly less than 10,000 Angstroms, indicating a possilbe connection between the planes favored in precipitation and the secondary net of planes whose spacing has been estimated as being of that order of magnitude in metals. It may be significant that, among a series of samples aged for various times at the optimum temperature, the first reasonably complete and measureable nets were found in grains of the sample aged for the time found by other investigators to be the optimum time of treatment for producing hardness in this alloy.

59. A Grating Interferometer. BERTRAND P. RAMSAY, *University of Kentucky. (Introduced by O. T. Koppius.)*—Discussions of the Michelson interferometer indicate that fringes are formed by the superposition of two approximately parallel wave fronts. Wave fronts having the necessary characteristics can be obtained by mounting gratings on the arms of the interferometer in the places of the customary reflecting mirrors. The instrument becomes its own monochromator when the gratings are rotated to positions of minimum deviation for some particular wavelength. The characteristics of the intensity patterns when the constants of the interfering beams are different have been studied. The customary form of the Michelson interferometer may be considered as the special case of the grating interferometer for which the grating space is infinite and the spectrum order is zero.

60. The Branching of Lightning and the Polarity of Thunderclouds. J. C. JENSEN, *Nebraska Wesleyan University.*—This is a continuation of investigations previously reported. In thundercloud observations extending over a period of four years 77 photographs have been obtained which show downward branching of lightning discharges from a negative cloud to ground while only 13 discharges to ground with downward branching came from positive clouds. Discharges from a positive cloud to ground are characterized by many more finely branched streamers than those from a negative cloud. This is in agreement with Allibone, and with Peek, whose laboratory experiments show branching from a negative to a positive, grounded plane as well as in the opposite direction, and with engineering reports that power-line outages due to lightning are almost always caused by discharges of negative electricity to ground. The meteoroligical and electrical data obtained in the storms studied show general agreement with the type of cloud described by Banerji in which there is a concentration of negative charge in the front portion.

61. An Experiment to Detect Photon Spin. RICHARD A. BETH, *Worcester Polytechnic Institute. (Introduced by A. Wilmer Duff.)*—There is a possibility that the angular momentum of circularly polarized light (photon spin) can be detected and measured directly by mechanical means. The following apparatus and procedure are being used for a preliminary test in our laboratory. A circular half-wave plate of mica, 3 cm in diameter, is suspended in a vacuum from a Wollaston wire 0.0005 cm in diameter and one meter long. A photon of right circularly polarized light passing vertically up through the half-wave disk emerges as left circularly polarized light and, according to theory should deliver an angular impulse of $2h/2\pi = h/\pi$ to the disk. Besides doubling the effect, this use of a half-wave plate has the very important advantage of nearly eliminating heating and radiometer effects on the disk. Assuming that the equivalent of 40 lumens of circularly polarized sodium light could be concentrated on the disk, and that a resonance method be used, it is shown that an amplitude of torsional oscillation of 1/100 radian could be built up. A much stronger beam of white light could be concentrated on the disk, and, even though only partially circularly polarized, would give a much stronger effect. This amplitude could easily be detected by the usual mirror and scale method.

62. Measurement of the X-Ray Absorption Coefficient of Xenon. T. N. WHITE, *Office of Cancer Investigations, U. S. Public Health Service, Harvard Medical School. (Introduced by B. E. Warren.)*—The absorption coefficient of xenon was measured through a wave-length range from 0.18 to 1.47A, with Soller slits allowing of an inhomogeneity of 0.04A. The results of two series of measurements under different conditions were quite consistent, and the values of log $(\mu/\rho - 0.70)$ lie on straight lines well within the errors of measurement both above and below the absorption limit. The values lie below those calculated from Jönsson's universal absorption curve, and in general the discrepancy is somewhat larger than would be expected.

63. Neutrons of High Energy from Cosmic-Ray Bursts in Aluminum. GORDON L. LOCHER, *National Research*

Fellow, Bartol Research Foundation of the Franklin Institute.—Cloud photographs in argon of the disintegration products of cosmic-ray Stösse, or ionization bursts, arising from aluminum, show recoil atom tracks similar to those of argon atoms recoiling from collisions with neutrons, but of very much larger *volume*. These are attributed to argon atoms that have been stripped of their extra-nuclear electrons by the sudden impact of *neutrons of very high energy*. W. F. G. Swann has pointed out that since the specific ionization of a charged particle is proportional to N^2, where N is the number of charges, such stripped recoil atoms would dissipate a large amount of kinetic energy in a very short path; also, that the *broadness* of the tracks is to be expected on the basis of the momentum transmitted to the electrons by collision with the highly-charged recoil nucleus. Thus the energy communicated by a particle of charge Ne is N^2 times that given by a particle of the same mass, and charge e. Hence all electrons ejected by a stripped argon nucleus may make tracks of appreciable lengths (similar to fast delta-rays), adding subsidiary ionization that extends to appreciable distances from the primary path. The ranges of these electron tracks are calculated, approximately, for a charge of $18e$, and are found to be in approximate agreement with the radius of the largest ionization cluster photographed, namely about 1.5 cm. The energies of the recoil atoms so far photographed in Al-Stösse are uniformly larger than those of the Pb-Stösse previously reported (Locher, Phys. Rev. **44**, 779 (1933)), which suggests that *characteristic differences exist between the processes giving rise to cosmic-ray neutrons from Stösse in different substances*. The paucity of electron tracks in photographs of Al-Stösse, as compared with those from lead, is also of interest.

64. The Raman Spectrum of Heavy Water. R. W. Wood, *Johns Hopkins University.*—The Raman spectrum of 18.5 percent heavy water has been obtained with excitation by the 2536 mercury line. A quartz tube 35 cm long with a capacity of 7.5 cu. cm was clamped to a Hannovia Hg vacuum tube of quartz and the whole surrounded by a cylinder of polished aluminum. The band of frequency difference 3420 due to ordinary water was photographed in twenty minutes with a density equal to that of the new band due to HOH^2 obtained with an exposure of 80 minutes. The frequency difference for this band was 2549 in good agreement with theory. The water was prepared by John W. Murray of the Chemistry Department.

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